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Non-linear model in region of very low speeds for a permanent magnet direct current motor

Nickolay Popov
Institute of Robotics
Bulgarian Academy of Sciences,
Plovdiv, Bulgaria
njpopov62@gmail.com

Abstract. **Thematic is in mechatronics and automation branches, applicable in the mobile robotics. Permanent Magnet DC collector motors, are widely used in small mobile robots due to their low-cost. Automated control systems of mobile robots, which operate under different conditions and require accuracy of operation, raise the need for the nonlinearities to be taken into account. In this article, a complex non-linear model of a PMDC motor with brushes is synthesized. The aim is to determine of suitable way of motor behaviour simulating in the region of very small speeds. The tribology aspects at different friction regimes are of great importance for a model at low speeds. The parameters and constants of the model are separately defined through referring to their physical equivalents. Besides the theoretical modelling, a simple mathematical way to determine the constants for this detailed model is deduced. Then the synthesized model is simulated and results are graphically represented and then compared with another similar model, proposed by another authors. As a conclusion, the advantages of this non-linear approach are revealed. This research is applicable as a study of direct-current motor and its simulation model or as facilitating example in lectures of robotics or control systems.**

Keywords: Mechatronics, Mobile robot, PMDC motor, Non-linear modelling, Tribology aspects, Torque at low rotor speed.

I. INTRODUCTION

In this report is studying the non- linear motor behaviour at very low speeds, where the effect of increased friction is observed. The topic is in the mechatronics and the control theory, because this effect causes the plant (the motor) to have a hysteresis in its output torque and rotational speed. The main goal is to study this non-linearity, so the management of motor speed and torque to be adequate when a closed- loop control is used. Question we pose is how to model the motor (plant) behaviour, so it to be continuous and analytically smooth in the region of this hysteresis. The importance of this question arises when we simulate a motor. The way this effect to be modelled determines the speed of simulation process.

The results are applicable in wide spectrum of electrical motors. The article is oriented to the class of PMDC (Permanent Magnet Direct Current) motors simulations, in the mobile robotics having speed reducer (gear) before the wheels, but the created math model is applicable for any other electrical motor to simulate.

The state of the art studies use same approach to represent this effect: Stribeck plus Coulomb friction plus viscous friction [1]. This is valid for a wide range of bearings, frictional joints, gears, etc. ad it is a definition for so called “Static model” [2], while the studies [3], [4] deal with “LuGre dynamic” model, extending the static model to account for hysteresis effects. In [5] is studied for the vehicle dynamics by application of both models. Motor drive systems with and without gear are examined in [6], while [7] gives some applications of both models in automation theory. The measurement needed for obtaining the static model parameters is given in [8], where the parameters of mathematical formulae are taken from the classical articles for static and dynamic models [9] and [10]. In field of electrical motors control, the closely related work is made in the [11], [12] and [13], where it is proposed to be used the static model of DC motor. In this article we will name it the “old Stribeck formula”. The adequacy "model to real motor" is suppose to be good in some degree, as it is predicted by the physics theory. The studies on this topic propose to improve the friction model by using the classical static model [9] with an additional parameter (sharpness factor). Although [6] and [9] study the sharpness factor in details, for now it has been not tested with PMDC motor. Latter friction model we test in present article and we name it „new Stribeck formula“.

This study advance our knowledge by creating a more precise static model for a PMDC motor, taking into account the differences in bearings and gear construction, lubrication method, etc. Our contribution to the topic is in the fact that the resulting model, although being more

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complicated, is more adequate to predicted by the theoretical physics: It allows more precisely to approximate the process behaviour. The increased lab work is just a bit more than previously proposed "old Stribeck approach".

The hypothesis is: The proposed in this article method is more versatile than that given in [11], [12] and [13], as it takes into account one parameter more: In particular this additional parameter, introduced in [6] and [9] allows us to work with different behaviours of non-linear friction, in the way it depends on the type of bearings, gear, etc.

This manuscript is organized in following parts: Introduction, Materials and methods, Results and discussion, Conclusions

II. MATERIALS AND METHODS

A. Brief Theory

The used physical symbols and abbreviations and their meanings are given in Table. 1.

a) Linear PMDC motor model:

It is given as two equations of balance: electrical and electro- mechanical as well as two equal motor constants, when condition of stationary (settled) angular speed is imposed (i.e. no speed and current variations):

$$v_a = R_a . i_a + K_b . \omega_m \quad (1)$$

$$K_m = K_b \quad (2)$$

$$K_m . i_a = B_m . \omega_m + T_{lin} \quad (3)$$

b) Stribeck-Coulomb friction model:

We will examine two equations about Stribeck-Coulomb friction torque

- *Old friction model:* Friction torque as proposed by other authors: In [11], [12] and [13] is introduced an additional friction torque T_{strib} in order to account for the increased friction at very low rotational speeds; This additional friction (called Stribeck Friction) represents the fact that at low velocity the friction consumes all the motor torque:

$$T_{strib}(\omega_m) = \alpha_0 + \alpha_1 . e^{-\alpha_2 . \omega_m} \quad (4)$$

- *New friction model:* It is proposed by this article: We propose the improvement of motor model by using the classical static model [1] with sharpness factor ν being not unit, as proposed in [6] and [9]:

$$T_{stib} = T_{kinetic} + T_{kinstat} . e^{-(\omega_m/\omega_{strib})^{\nu}} \quad (5)$$

The meaning of used coefficients in both formulae (4) and (5) is as follows:

$\alpha_0 = T_{kinetic}$ represents the kinetic (Coulomb) friction, which is a constant with dimension of $[N.m]$; $\alpha_1 = T_{kinstat}$ is the kineto-static (Stribeck) constant, representing the friction torque at zero speed. Its dimension is $[N.m]$; $\alpha_2 = 1/\omega_{strib}$ represents so called Stribeck critical speed ω_{strib} in following manner: below this speed the additional friction effect becomes significant and vice versa, above this critical speed the effect is negligible; Dimension for α_2 is $[sec/rad]$. The sharpness factor ν for the exponential term is introduced in order to represent the "transition sharpness" at critical speed for the effect appearing/ disappearing: The dimensionless values of $\nu = 0.5 - 2$ are possible, the lower limit is for sleeve bearings with cheap gear, i.e. light "transition sharpness"; The upper limit is for ball bearings with a really good gear, thus "sharpness" is to be great, according [1], [6] and [9]. In the formulation of additional friction torque as proposed by other authors [11], [12] and [13], a default (immediate) value of $\nu = 1$ is put on.

c) Values of kineto-static and kinetic constants:

Values of ω_{strib} and ν have meaning of controlling

TABLE 1 USED PHYSICAL SYMBOLS AND ABBREVIATIONS

Symbol	Meaning and role of parameter	Dimens.
Motor parameters, given by manufacturer		
v_a	Armature voltage	[V]
T_{stall}	Stall torque[1]	[N.m]
ω_{noload}	noload speed[1]	[rad/s]
Calculated constants for motor equations		
R_a	Armature resistance of rotor	[Ohms]
K_b	Back-EMF constant for motor	[V/rad/s]
K_m	Torque constant for motor	[N.m/A]
i_{noload}	noload current	[A]
B_m	Viscous friction coefficient	[N.m.s/rad]
Independent variable, used in motor and Stribeck laws		
ω_m	Rotational speed of the rotor	[rad/s]
Dependant values, obtained by linear motor law		
i_a	Armature current	[A]
T_{lin}	Output torque when linear law	[N.m]

TABLE 1 USED PHYSICAL SYMBOLS AND ABBREVIATIONS (CONTINUED)

Symbol	Meaning and role of parameter	Dimens.
Independent parameters, used in experiments of Stribeck law		
ω_m	Critical Stribeck speed[1]	[rad/s]
α_2	Reciprocal of Stribeck speed[2]	[s/rad]
ν	Stribeck sharpness factor[1]	[-][3]
Dependant values, constrained by Stribeck limit conditions		
$T_{kinetic}$	Kinetic torque[1]	[N.m]
α_0	Kinetic torque	[N.m]
$T_{kinstat}$	Kineto- static torque[1]	[N.m]
α_1	Kineto- static torque[2]	[N.m]

Dependant values, obtained by Stribeck law		
T_{strib}	Stribeck torque	[N.m]
κ_{strib}	Stribeck losses factor	[-][3]
T_{final}	Output torque when Stribeck effect	[N.m]
Dependant values, used in Criterion calculations		
Ω_{loss}	Relative speed at given % of losses	[-][3]
$\Omega_{loss,old}$	Relative speed at % losses by old method	[-][3]
$\Omega_{loss,min}$	Relative speed at % losses by new method, minimal value	[-][3]
$\Omega_{loss,max}$	Relative speed at % losses by new method, maximal value	[-][3]
$\Delta_{loss,min}$	Relative speed minimum at % loss for new method, compared to the old one	[-][3]
$\Delta_{loss,max}$	Relative speed maximum at % loss for new method, compared to the old one	[-][3]

1 Notations used in [6], [9] and presented article
2 Notations used in referenced articles, e.g. [11], [12] and [13]
3 Dimensionless

values for the friction model, i.e. they are chosen by the experimenter in order to best fit the Stribeck function in two points in the new proposed model (according [6], [9] and as used in this article); Or just in one point, as it is in the old model proposed by [11], [12] and [13], where in this case $\nu = 1$ is fixed.

At this choice of ω_{strib} and ν the resulting Stribeck torque value must satisfy two physical conditions:

a) At stall the Stribeck torque to be the same as the motor stall torque:

$$T_{strib}\big|_{\omega_m=0} = T_{stall} \quad (6)$$

b) At noload the Stribeck torque has to absent:

$$T_{strib}\big|_{\omega_m=\omega_{noload}} = 0 \quad (7)$$

This gives us two constrains (coherency conditions), by which the values of $T_{kinetic}$ and $T_{kinstat}$ have to dependent on the chosen ω_{strib} and ν :

$$T_{kinetic} = \frac{T_{stall} \cdot e^{-(\omega_{noload}/\omega_{strib})^{\nu}}}{e^{-(\omega_{noload}/\omega_{strib})^{\nu}} - 1} \quad (8)$$

$$T_{kinstat} = T_{stall} - T_{kinetic} \quad (9)$$

d) *Output torque with Stribeck-Coulomb friction:*

At some speed ω_m the "ideally delivered" in the output is the torque T_{lin} , as stated by the linear motor law. In the reality the delivered torque is decreased by the value of Stribeck torque T_{strib} , so finally we have:

$$T_{final} = T_{lin} - T_{strib} \quad (10)$$

e) *Stribeck losses factor:*

We introduce Stribeck losses factor κ_{strib} the similar way the Efficiency concept is defined i.e. "(output power when Stribeck law present) / (output power when pure linear motor law)" :

$$\kappa_{strib} = \frac{(T_{lin} - T_{strib}) \cdot \omega_m}{T_{lin} \cdot \omega_m} = 1 - \frac{T_{strib}}{T_{lin}} \quad (11)$$

It is clear that Stribeck losses factor is inside the unit range: $0 \le \kappa_{strib} \le 1$ or $0\% \le \kappa_{strib}[\%] \le 100\%$.

Also, we interest what the speed is when a specified loss value $\kappa_{strib} = \kappa_{loss}$ occurs. This speed is important when to compare both Stribeck models. It is also convenient to be converted in relative form, i.e. compared to speed at noload ω_{noload} , by introducing:

$$\Omega_{loss} = \frac{\omega_m}{\omega_{noload}}\bigg|_{\kappa_{strib}=\kappa_{loss}} \quad (12)$$

It is clear that this is inside the unit range: $0 \le \Omega_{loss} \le 1$ or $0\% \le \Omega_{loss}[\%] \le 100\%$.

f) *Hypothesis:*

The new Stribeck equation gives more approximation freedom in the region of interpolation, compared to the old formula. The new equation will be analysed as some kind of "deviation" around the values given by the old one.

g) *Criterion:*

We will compare the new Stribeck equation and the old one, by observing their speed values at same losses factor. The old equation with fixed ω_{strib} has the fixed relative speed $\Omega_{loss,old}$ at some level of losses. The new Stribeck formula allows the relative speed to vary in the range $\Omega_{loss,min}$ to $\Omega_{loss,max}$, located around the old value; It is due to varying the sharpness factor in range $\nu_{min} \leqslant \nu \leqslant \nu_{max}$ even at same fixed ω_{strib} .

The Criterion for „flexibility" (freedom) of the new equation is given as a kind of relative amplitude variation of values, compared with the old equation:

$$\Delta_{loss,min} = \frac{\Omega_{loss,min} - \Omega_{loss,old}}{\Omega_{loss,old}} \quad (13)$$

$$\Delta_{loss,max} = \frac{\Omega_{loss,max} - \Omega_{loss,old}}{\Omega_{loss,old}} \quad (14)$$

We will examine these values and give them in form of percentages.

B. Implementation

The calculations has been done in Octave© [17], with the following algorithm has been used:

a) Given motor data (e.g. by manufacturer): These are v_a , i_{stall} , T_{stall} , ω_{noload} .

b) Calculation of some motor parameters is done the same way as in author's previous article [14]:

$$R_a = \frac{v_a}{i_{stall}} \quad (15)$$

$$K_b = K_m = \frac{T_{stall}}{i_{stall}} \quad (16)$$

$$i_{noload} = i_{stall} - \frac{T_{stall}}{v_a} \cdot \omega_{noload} \quad (17)$$

$$B_m = \frac{T_{stall}}{\omega_{noload}} \cdot \frac{i_{noload}}{i_{stall}} \quad (18)$$

c) The linear motor behaviour is obtained in the same way as in [14], when the motor speed gradually changes in range $[0 - \omega_{noload}]$:

$$i_a = \frac{v_a - K_b.\omega_m}{R_a} \quad (19)$$

$$T_{lin} = K_b . i_a - B_m . \omega_m \quad (20)$$

d) Choose some values for Stribeck critical speed and sharpness factor, such that $0 \leq \omega_{strib} \leq \omega_{noload}$ and $0.5 \leq \nu \leq 2$, for calculation and experiment purposes.

e) Calculate the values of kinetic and kineto-static constants for each chosen ω_{strib} and ν :

$$T_{kinetic} = \frac{T_{stall} . e^{-(\omega_{noload}/\omega_{strib})^{\nu}}}{e^{-(\omega_{noload}/\omega_{strib})^{\nu}} - 1} \quad (21)$$

$$T_{kinstat} = T_{stall} - T_{kinetic} \quad (22)$$

f) Calculate the value of Stribeck- Colomb friction value for each chosen ω_{strib} and ν :

$$T_{stib} = T_{kinetic} + T_{kinstat} . e^{-(\omega_m/\omega_{strib})^{\nu}} \quad (23)$$

g) Calculate the output torque when the Stribeck effect is present by means of obtained Stribeck values:

$$T_{final} = T_{lin} - T_{strib} \quad (24)$$

h) Calculate Stribeck losses factor by means of:

$$\kappa_{strib} = 1 - \frac{T_{strib}}{T_{lin}} \quad (25)$$

Now we can recalculate the Stribeck losses factor in percentages: $0\% \leq \kappa_{strib}[\%] \leq 100\%$.

i) Find out the speed where a predefined friction value $\kappa_{strib} = \kappa_{loss}$ is reached. Then calculate the relative value of this speed in respect to noload speed ω_{noload} :

$$\Omega_{loss} = \left. \frac{\omega_m}{\omega_{noload}} \right|_{\kappa_{strib} = \kappa_{loss}} \quad (26)$$

Do this for both Stribeck equations (old and new one), this way calculating the values of $\Omega_{loss,old}$ for the old Stribeck formulation, as well as $\Omega_{loss,min}$, $\Omega_{loss,min}$ for the new equation. Then convert them in percentages.

j) Calculate the criterion for „flexibility" for the new equation as relative variation of values, compared to these with the old equation:

$$\Delta_{loss,min} = \frac{\Omega_{loss,min} - \Omega_{loss,old}}{\Omega_{loss,old}} \quad (27)$$

$$\Delta_{loss,max} = \frac{\Omega_{loss,max} - \Omega_{loss,old}}{\Omega_{loss,old}} \quad (38)$$

Now convert these values in form of percentages.

C. Notes and examples

The main problem is in the fact, we must not to run a motor in the extremely low speeds, with high loading torque, as it will cause the gear and bearings to overheat, or the rotor and gear axes to break up. The one value for the phenomenon of friction increasing (i.e. first approximation point) can be measured at the low speed limit, recommended by the manufacturer. The other point can be chosen strongly inside the allowed speed range, where the Stribeck-Coulomb effect has low, but distinctive value.

Thus we will use the following relations, in the way these are predetermined in engineering practice. We interest where the Stribeck effect has reached some values, let's say as they are defined in [15]: $\kappa_{95} = 0.95$ - below this value the Stribeck effect is negligible in practice; $\kappa_{90} = 0.1$ - above this value we have clearly visible Stribeck torque; $\kappa_{50} = 0.5$ - above this value a great part of output torque is consumed by the bearings and gear, thus overheating them; Usually, for real PMDC motors these relative points can vary in ranges [15]: $\Omega_{90} = 0.1 - 0.2$ - visible Stribeck effect is below 10 to 20 % of noload torque; $\Omega_{50} = 0.05 - 0.15$ - Stribeck effect causes overheating below 5 to 15 % of noload torque; Such defined ranges of relative speed gives us reasonable limits where the influence of parameters controlling Stribeck torque to be investigated in the following experiments.

Thus in our experiments we will check these values for several points of speed, at several Stribeck losses factors: $\kappa_{strib} = 50\%, 90\%, 95\%$ and the respective relative speeds are $\Omega_{50}, \Omega_{90}, \Omega_{95}$. For case of new equation these relative speeds will vary, when varying ν: respectively $\Omega_{50,min}$ to $\Omega_{50,max}$, $\Omega_{90,min}$ to $\Omega_{90,max}$ and $\Omega_{95,min}$ to $\Omega_{95,max}$. The old equation is fixed to $\Omega_{50,old}, \Omega_{90,old}, \Omega_{95,old}$ because of fixed $\nu = 1$.

The linear model parameters are taken from [14] for a real motor data [16]. It is a PMDC brushed motor with cylindrical gear and sleeve bearings.

a) Manufacturers motor data [16] are: v_a=12[V], i_{stall}=10[A], T_{stall}= 29.8[N.m] and ω_{noload}=2.41[rad/sec].

b) Calculated motor parameters: R_a=1.2[Ohms], K_b=K_m=2.98[V/rad/sec] or [N.m/A], $B_{\_m}$=4.9648 [N.m.sec/rad], i_{noload}=4.0152[A].

c) Calculated linear (ideal) motor behaviour: the angular speed changes in range ω_m=[0↔2.41] [rad/sec]. Dependent values change are: i_a=[10↔ 4.0152] [A], T_{lin}=[29.8↔0] [N.m].

d) Calculating the change of Stribeck caused losses at fixed ω_{strib}=0.2 [rad/sec] and ν=1[-]: The values of kinetic and kineto-static constants are: $T_{kinstat}$=-1.7417x10^{-4}[N.m] and $T_{kinetic}$=29.8[N.m]. When the angular speed changes ω_m=[0↔2.41] [rad/sec], the Stribeck losses: T_{strib}=[29.8↔0] [N.m] and κ_{strib}=[1↔0] [dimensionless], respectively.

e) Calculation of Stribeck losses at fixed ν=1[-] and different ω_{strib}=0.5; 0.25; 0.125; 0.0625 [rad/sec]: The values of kinetic and kineto-static constants are: $T_{kinstat}$=30.042; 29.802; 29.8; 29.8 [N.m] and $T_{kinetic}$=-0.24235; -0.0019393; -1.2619x10^{-7}; -5.3434x10^{-16} [N.m] and dependent values change as: T_{stib}=[29.8↔0] [N.m] and κ_{strib}=[1↔0] [-], respectively.

f) Calculation of Stribeck caused losses at fixed ω_{strib}=0.2[rad/sec] and different ν = 0.5; 0.75; 1; 1.5; 2[-] : The values of kinetic and kineto-static constants are respectively: $T_{kinstat}$=29.872; 29.804; 29.8; 29.8; 29.8[N.m] and $T_{kinetic}$=-0.072218; -0.0035427; -0.00017417; -4.2106x10^{-7}; -1.0179x10^{-9} [N.m].

g) Finding the values at κ_{90}=0.5=50% is done by changing the values for ω_m=[0-2.41][rad/sec], ν=[0.5-2]. Finding the values at κ_{90}=0.9=90% and κ_{95}=0.95=95% is done by the same technique.

i) Finding the values at is done by the same way.

For each calculation: some check speeds are chosen and their calculated values are given in Fig.1, Fig.2 and Fig.3. The following results for the sharpness factor ν_{sample}=1 (old method approach) for different values of ω_{strib} are obtained as given in Fig.4, Fig.5 and Fig.6.

III. RESULTS AND DISCUSSION

A. Experiments

a) *Geometrical interpretation of Stribeck losses factor:* the goal is to show how the Stribeck torque affects the output torque and to find the Stribeck losses factors at several motor speeds: Fig.1 gives a plot of Stribeck caused losses at fixed ω_{strib} and ν : The "linear motor behaviour torque" is given too. The Stribeck torque (causing the non- linear "deformation" of output torque) is maximum at zero speed, as resulting torque at motor stall becomes 0; And at contrary it becomes negligible at high speeds, i.e. it tends to zero when getting close to noload speed, and resulting curve converges to the line of linear motor behaviour. Thus curve confirms the expected behaviour for this type of friction at low speeds.

b) *Controlling Stribeck friction when different Stribeck critical speeds are used at fixed sharpness factor:* Fig.2 shows some plots of Stribeck losses at fixed fixed sharpness factor ν when different Stribeck critical speeds ω_{strib} are applied and how the Stribeck torque affects the output torque and allows to find how the Stribeck losses factor changes when Stribeck critical speed changes. The graph shows that the non- linear "deformation" is controllable by chosen Stribeck critical speed values only in the region close to zero motor speed, while the "deformation" is negligible at noload speed. This type of function control is available in all compared articles: [11], [12], [13] (old used technique) control the Stribeck curve only in this way. [1], [6], [9] and this article can change the curve by this way too.

c) *Controlling Stribeck friction by different values of sharpness factor at fixed Stribeck critical speed:* Fig.3 gives us some plots of Stribeck losses at fixed ω_{strib} at different ν and it shows the way the Stribeck losses factor changes when sharpness factor changes. The resulting curve is well controlled close to maximum torque; Non-linearity is negligible near maximum speed. This applies only for [6], [9] and this article; And not for [11], [12], [13] where sharpness factor is fixed.

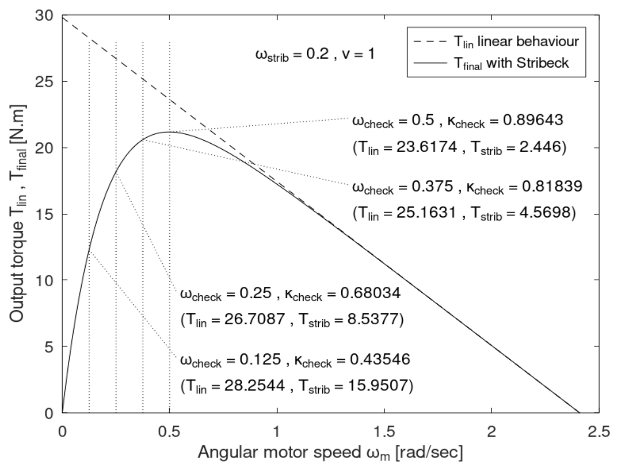

Fig. 1. Plot of Stribeck caused losses at fixed Stribeck critical speed ω_{strib} and fixed sharpness factor ν .

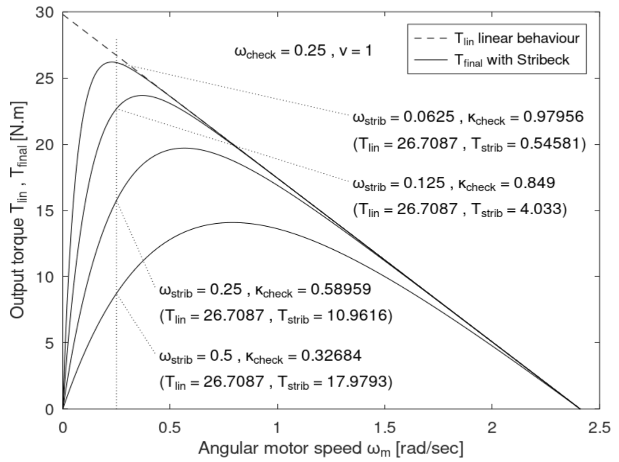

Fig. 2. Plot of Stribeck caused losses at fixed sharpness factor ν and different Stribeck critical speeds ω_{strib} .

d) *Speed deviations ("freedom") available by the new method, compared to the old one, when losses factor is 50% :* The question is: can the new Stribeck formulation to embrace all the real cases, i.e. is the range of relative speed Ω_{50} (where losses factor is 50%) wider than the manufacturers data, cited above. This is shown in Fig.4 : The abscissa is the sharpness factor, the ordinate is this relative speed. The family of curves is given for several fixed Stribeck critical speeds. The vertical line $\nu = 1$ represents the values obtained by the old Stribeck formulation; The shown points has Y-value Ω_{50} , representing the old formula. The curves itself represented the change of obtained relative speeds by

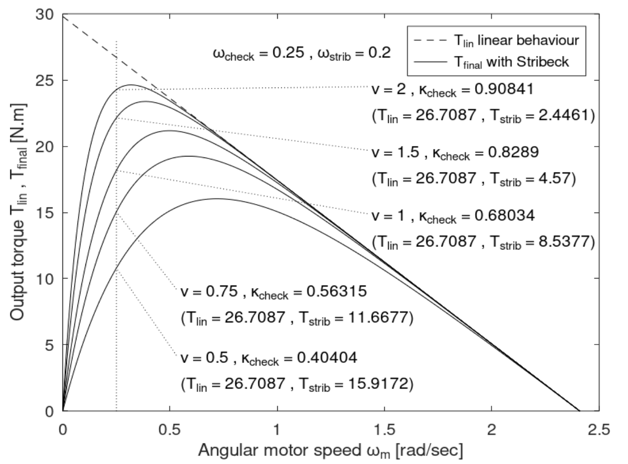

Fig. 3. Plot of Stribeck caused losses at fixed Stribeck critical speed ω_{strib} and different sharpness factors ν.

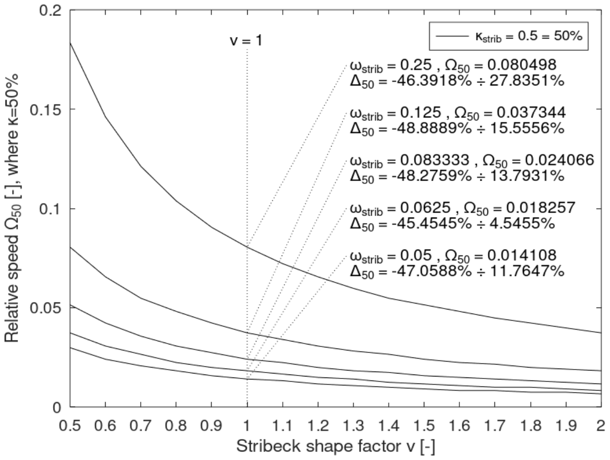

Fig. 4. Plot of speeds ω_{50} when Stribeck losses are $\kappa_{50} = 50\%$, but given as relative speed $\Omega_{50} = \omega_{50}/\omega_{noload}$ at different Stribeck critical speeds ω_{strib} and different sharpness factors ν .

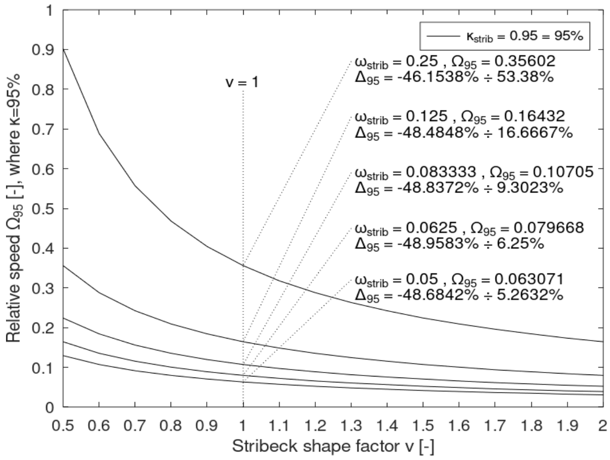

Fig. 6. Plot of Stribeck losses $\kappa_{95} = 0{,}95\%$ appearance at some speed ω_{95} , but given as relative speed $\Omega_{95} = \omega_{95}/\omega_{noload}$ at different Stribeck critical speeds ω_{strib} and different sharpness factors ν .

the new Stribeck equation; The values of Δ_{50} reveal the possible deviations obtained by the new equation. The range -48% to +27% shows we have a good "freedom" for the new method, compared with the old one.

e) *Speed deviations available by the new method, compared to the old one, when losses factor is 90% :* Again, the question is: can the new Stribeck formulation embrace a relative speed range Ω_{90} (where losses factor is 90%) wider than the manufacturers data. This range is given in Fig.5 : Abscissa is the sharpness factor while the ordinate is this relative speed. Curves family is for several Stribeck critical speeds. The vertical line $\nu = 1$ shows the values obtained by the old Stribeck equation; The shown Ω_{90} represent the results by the old formula. The curves also show the range of obtained relative speeds by the new method [6], [9] and this article; The values of Δ_{90} reveal the deviation accessible by the new equation is -50% to +42% , i.e. good "flexibility" for the new method, compared with the old one.

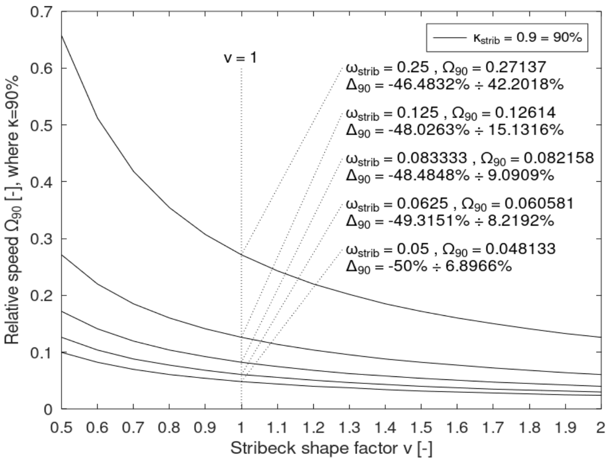

Fig. 5. Plot of Stribeck losses $\kappa_{90} = 90\%$ appearance at some speed ω_{90} , but given as relative speed $\Omega_{90} = \omega_{90}/\omega_{noload}$ at different Stribeck critical speeds ω_{strib} and different sharpness factors ν .

f) *"Freedom" obtainable by the new method, compared to the old one, when losses factor is 95% :* The modelling range in high- speed region, where the Stribeck friction losses are to be negligible (in this case 5%) is shown in Fig.6 as relative speed Ω_{95} (where losses factor is 95%). The designation and meanings are the same as explained above. The range is -50% to +42%, but all methods [6], [9], [11], [12], [13] and the presented here, have the property that the Stribeck effect to disappear at high relative speeds (values between 0.1 and 0.65): Thus the new simulation model is applicable.

B. Discussions

The „percentage of freedom“ used as criterion has range of nearly ±50%, by the new introduced sharpness factor and this proves the hypothesis. At high motor speeds the influence of the sharpness factor can be neglected.

IV. Conclusions

The proposed new Stribeck formula gives us a more flexible choice of static friction modelling, when types of motor bearings and gear differ. Having only two parameters ω_{strib} , ν , we can preliminary construct the new static friction model by measurements as explained in [8]; Or these parameters can be identified “in real time” by methods given in [5] and [9]. And finally, the new static friction model with two control parameters can be used even for some simplified preliminary engineering designing, using only the manufacturer’s data (usually the scarce ones), without experiments.

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