Two Generalizations of Skew-Symmetric Sequences With Odd Lengths

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Abstract. **The signals, exploited by the radar sensor networks and remote control systems, have to provide simultaneously high range resolution and ability to work stable in a hostile radio electronic environment. An effective approach for satisfying of these requirements is the frequent change of many different signals, which autocorrelation functions have small sidelobes. Accounting this situation in the paper the generalizations of the skew-symmetric sequences with odd lengths, which are phase manipulated signals, possessing high autocorrelation merit factor, are explored. As a result, two methods for synthesis of infinite families of phase manipulated signals with good autocorrelation properties are substantiated.**

Keywords: skew-symmetric sequences, synthesis of signals with small autocorrelation.

I. INTRODUCTION

Today the so-called Internet of things (IoT) is one of the basic system of technologies, providing the rapid progress of the Fourth Industrial Revolution ("4IR", or "Industry 4.0"), which will improve radically the life-style of all people around the world [1]. The advancement of the IoT but essentially depend on the ability to provide the electromagnetic compatibility of a great amount of specialized small radar sensor networks and remote control systems, operating simultaneously, as well as their stable performance in a hostile radio electronic environment [2].

As known, an effective approach for satisfying of these requirements is the frequent change of many different signals, which autocorrelation functions have small sidelobes [3], [4]. Accounting this situation in the paper the generalizations of the so-called skew-symmetric sequences with odd lengths, which are phase manipulated (PM) signals, possessing high autocorrelation merit factor, are explored. As a result, two methods for synthesis of infinite families of PM signals with good autocorrelation properties are substantiated.

The paper is organized as follows. First, the basics of the signal processing in the receivers of the communication systems are recalled. After that the possibility the negative effects, caused by the multipath spread of the electromagnetic waves, to be minimized by the application of PM signals with good autocorrelation properties are investigated. On this base two methods for synthesis of infinite families of PM signals, which are skew-symmetric sequences with odd lengths, possessing high autocorrelation merit factor, are substantiated. At the end, the applications of the proposed PM signals are analysed.

II. METHODOLOGY OF THE STUDY

Most often, the digital signal processing in the receivers of radio-communication systems (RCSs) can be described by the following polynomial model [5]:

$$\left[\sum_{k=0}^{N-1} \mu(k)x^{i}\right]\left[\sum_{k=0}^{N-1} \mu^{*}(k)x^{-i}\right] = \\ = \sum_{r=-N+1}^{N-1} P_{\mu\mu}(r)x^{r} \ . \qquad (1)$$

In (1) the following notations are used. First, the digital signal:

$$\{\mu(k)\}_{k=0}^{N-1} = \{\mu(0), \mu(1), \dots, \mu(N-1)\}, \qquad (2)$$

consists of the samples of the digitalized received signal. As the sent signal passes a large distance between the transmitter and the receiver the received signal is a diminished copy of the signal, emitted by the transmitter [3], [4], [5]. Due to this reason, the samples (2) are complex numbers, presenting the complex envelopes of the consecutive elementary phase symbols (or chips) with duration τ_{ch}, forming the sent signal.

Analogously, the sequences of complex numbers:

$$\{\mu^{*}(k)\}_{k=0}^{N-1} = \{\mu^{*}(0), \mu^{*}(1), \dots, \mu^{*}(N-1)\}, \qquad (3)$$

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$$\{P_{\mu\mu}(r)\}_{r=-N+1}^{N-1} = \{P_{\mu\mu}(-N+1), \dots, \\ \dots, P_{\mu\mu}(-1), P_{\mu\mu}(0), \dots P_{\mu\mu}(N-1)\}, \quad (4)$$

are the transfer function's (TF) samples of the finite impulse response (FIR) matched filter (FIRMF), used in the receiver, and the autocorrelation function (ACF) of the digital signal (2) respectively.

In (3) the symbol "*" stands for "complex conjugation".

Second,

$$F_{\mu}(x) = \sum_{k=0}^{N-1} \mu(k)x^{i}, \quad (5)$$

is the so-called generating function or Hall polynomial, associated with the digital signal (2).

It should be pointed out that the powers of the variable x^{-k}, x^{k} denote "overtaking or delay at k time-clocks with duration τ_{ch} during the signal processing" respectively.

Analogously, the generating functions (Hall polynomials) of the digital signals (3) and (4) are

$$F_{\mu}^{*}(x^{-1}) = \sum_{k=0}^{N-1} \mu^{*}(k)x^{-i}, \quad (6)$$

$$F_{P_{\mu\mu}}(x) = \sum_{r=-N+1}^{N-1} P_{\mu\mu}(r)x^{r}. \quad (7)$$

From (1), (2), (3), (4), (5), (6) and (7) it is not hard to obtain the next well known formula for direct calculation of the ACF's samples:

$$P_{\mu\mu}(r) = \\ = \begin{cases} \sum_{k=0}^{N-1-\lceil r \rceil} \mu(k+\lceil r \rceil)\mu^{*}(k), -(N-1) \le r \le 0, \\ \sum_{k=0}^{N-1-r} \mu(k)\mu^{*}(k+r), \quad 0 \le r \le N-1. \end{cases} \quad (8)$$

Now the following two facts should be taken in consideration.

First, in order to diminish the negative effects, caused by the multipath spread of the electromagnetic waves, the ACF of the sent signal have to resemble a delta pulse (i.e. to have thumbtack form) [3], [4], [5].

Second, the simplest type of phase manipulation, which is the most preferable for applications in the small (miniature) radio-electronic devices, is the binary phase shift keying (BPSK). When it is exploited the samples of the digital signal (2) can be considered as 1 and -1.

From the above facts it can be concluded that the ideal ACF of binary signals with crest factor equal to one has the form:

$$P_{\mu\mu}(r) = \begin{cases} N, N-\lceil r \rceil = N, \\ \pm 1, N-\lceil r \rceil = odd, \\ 0, N-\lceil r \rceil = even. \end{cases} \quad (9)$$

The binary signals with crest factor equal to one, which ACFs satisfy (9), are called Barker codes or Barker sequences after their inventor Ronald Hugh Barker.

Accounting the ideal autocorrelation properties of the Barker codes (signals), they will be analyzed in more detail in the sequel in this chapter of the paper.

More specifically, ever the generating function (Hall polynomial), associated with the digital signal (2), can be presented as a sum of two polynomials, comprising only the even and odd powers of the variable x respectively:

$$F_{\mu}(x) = F_{e\mu}(x) + F_{o\mu}(x), \\ F_{e\mu}(x) = \sum_{k=0}^{L} \mu(2k)x^{2k}, \\ F_{o\mu}(x) = \sum_{k=1}^{L} \mu(2k-1)x^{2k-1}. \quad (10)$$

Obviously, analogous separation of the generating function (Hall polynomial) (6), associated with the TF of the FIRMF, exploited in the receiver, is possible:

$$F_{\mu}^{*}(x^{-1}) = F_{e\mu}^{*}(x^{-1}) + F_{o\mu}^{*}(x^{-1}), \\ F_{e\mu}^{*}(x^{-1}) = \sum_{k=0}^{L} \mu^{*}(2k)x^{-2k}, \\ F_{o\mu}^{*}(x^{-1}) = \sum_{k=1}^{L} \mu^{*}(2k-1)x^{-2k+1}. \quad (11)$$

After taking into consideration (10) and (11) in (1), the result is:

$$\sum_{r=-N+1}^{N-1} P_{\mu\mu}(r)x^{r} = \\ = [F_{e\mu}(x) + F_{o\mu}(x)][F_{e\mu}^{*}(x^{-1}) + F_{o\mu}^{*}(x^{-1})] = \\ = F_{e\mu}(x)F_{e\mu}^{*}(x^{-1}) + F_{e\mu}(x)F_{o\mu}^{*}(x^{-1}) + \\ + F_{o\mu}(x)F_{e\mu}^{*}(x^{-1}) + F_{o\mu}(x)F_{o\mu}^{*}(x^{-1}). \quad (12)$$

Here it should be especially noted that during the analysis of Barker signals with odd lengths

$$N = 2L + 1, \quad (13)$$

Marcel Golay has observed [6], [7], [8], [9], that they satisfy the condition:

$$\mu(L+k) = (-1)^{k}\mu(L-k), \ \ k = 1, 2, \dots, L. \quad (14)$$

In order to reach the aim of this paper, the condition (14) will be generalized as follows:

$$\mu(L+k) = (-1)^{k}\mu^{*}(L-k), \ \ k = 0, 1, 2, \dots, L. \quad (15)$$

Due to the alternate symmetry of (14), the PM signals, which satisfy the condition (14), are called skew-symmetric sequences (signals). Analogously, in this paper when the condition (15) takes place the respective PM signal will be called generalized skew-symmetric (GSS) sequence (signal).

Here it should be especially noted that M. Golay has pointed out the skew-symmetric sequences as signals with good autocorrelation properties as for them the ratio of the mean peak energy to the aggregated energy of all the sidelobes:

$$MF = \frac{N^{2}}{2\sum_{k=1}^{N-1} \mu(k)\mu^{*}(k)} \quad (16)$$

is great [6], [10]. The ratio (16) is called the Merit Factor (MF) and together with the so-called Peak-to-Sidelobe ratio (PSR)

$$PSR = \frac{N}{\max_{k}\{|\mu(k)|\}_{k=1}^{N-1}} \quad (17)$$

are the two basic parameters for estimating the autocorrelation properties of every concrete PM signal. More specifically, the bigger are the MF and PSR, the better is the resolution of signals, passed different ways among the transmitter and the receiver.

From all the above stated, the following conclusions ensue.

First, the condition (15) simplifies to the condition (14), when in the communication devices only BPSK is used.

Second, if a PM signal satisfies the condition (15), then its ACF has non-zero sidelobes only when time-shifts r are even integers. Indeed, when $N \equiv 1mod4$, then $L \equiv 0mod2$. As a result $F_{e\mu}(x)$ contains odd (namely $(L+1)$) number of monomials and its reciprocal polynomial $\tilde{F}_{e\mu}(x)$ (i.e. the polynomial, which coefficients are arranged in the mirror reversal order $\{\mu(L-2k)\}_{k=0}^{L}$) has the form

$$\tilde{F}_{e\mu}(x) = \sum_{k=0}^{L} \mu(L-2k)x^{2k} = -F_{e\mu}^{*}(x). \quad (18)$$

Besides, $F_{o\mu}(x)$ contains even (namely (L)) number of monomials and its reciprocal polynomial $\tilde{F}_{o\mu}(x)$ is

$$\tilde{F}_{o\mu}(x) = \sum_{k=0}^{L} \mu(L-2k)x^{2k} = F_{o\mu}^{*}(x). \quad (19)$$

When $N \equiv 3mod4$, then $L \equiv 1mod2$ and, at the one hand, $F_{e\mu}(x)$ contains even number (namely $(L+1)$) of monomials and its reciprocal polynomial $\tilde{F}_{e\mu}(x)$ has the form

$$\tilde{F}_{e\mu}(x) = \sum_{k=0}^{L} \mu(L-2k)x^{2k} = F_{e\mu}^{*}(x). \quad (20)$$

At the other hand, $F_{o\mu}(x)$ contains odd number (namely (L)) of monomials and its reciprocal polynomial $\tilde{F}_{o\mu}(x)$ has the form

$$\tilde{F}_{o\mu}(x) = \sum_{k=0}^{L} \mu(L-2k)x^{2k} = -F_{o\mu}^{*}(x). \quad (21)$$

From (18) and (19) it ensues, that in the cases, when $N \equiv 1mod4$, the second polynomial product in (12) can be transformed as follows:

$$F_{e\mu}(x)F_{o\mu}^{*}(x^{-1}) = \tilde{F}_{e\mu}(x^{-1})\tilde{F}_{o\mu}^{*}(x) = \\ = \left[-F_{e\mu}(x^{-1})\right]^{*}F_{o\mu} = -F_{o\mu}(x)F_{e\mu}^{*}(x^{-1}). \quad (22)$$

Analogously, from (20) and (21) it ensues, that in the cases, when $N \equiv 3mod4$, the second polynomial product in (12) can be transformed as follows:

$$F_{e\mu}(x)F_{o\mu}^{*}(x^{-1}) = \tilde{F}_{e\mu}(x^{-1})\tilde{F}_{o\mu}^{*}(x) = \\ F_{e\mu}^{*}(x^{-1})\left[-F_{o\mu}^{*}(x)\right]^{*} = -F_{o\mu}(x)F_{e\mu}^{*}(x^{-1}). \quad (23)$$

After accounting (22) and (23) in (12), the result is:

$$\sum_{r=-N+1}^{N-1} P_{\mu\mu}(r)x^{r} = \\ = \left[F_{e\mu}(x) + F_{o\mu}(x)\right]\left[F_{e\mu}^{*}(x^{-1}) + F_{o\mu}^{*}(x^{-1})\right] = \\ = F_{e\mu}(x)F_{e\mu}^{*}(x^{-1}) + F_{o\mu}(x)F_{o\mu}^{*}(x^{-1}), \quad (24)$$

which has to be proven.

The above analysis demonstrates that the GSS signals could have ACFs, similar to the ideal ACF of binary signals (9), if the ACF sidelobes for even time-shifts are as small as possible. This problem will be studied in more detail in the next chapter of the paper.

III. RESULTS AND DISCUSSION

As mentioned above, a very effective approach for providing electromagnetic compatibility of all working simultaneously electronic devices as well as their resistance in a hostile radio electronic environment is the pseudo random change the emitted signals, using all volume of a large signal family. Accounting this fact in sequel the method, substantiated in the previous chapter of the paper, will be elaborated. This will allow two infinite families of GSS signals with odd lengths and nearly ideal ACF to be synthesized.

Obviously, the law of phase modulation (15), exploited for PM signal generation (or simply the coding rule (15)), is a generalization of skew-symmetric sequences with odd lengths, as the classic coding rule (14) is its particular case.

The reasonability of including of such GSS signals in the signal family, exploited by the communication devices, will be demonstrated by the means of the GSS signal with length $N = 15$:

$$\{\mu(k)\}_{k=0}^{14} = \{-1,-1,-1,-i,-i,-1,i, \\ i,-i,1,i,-i,1,1,-1\}, \quad i = \sqrt{-1}. \quad (25)$$

The ACF of the GSS signal (25) is presented on Fig. 1. As seen, the ACF of the GSS signal (25) has the form (9).

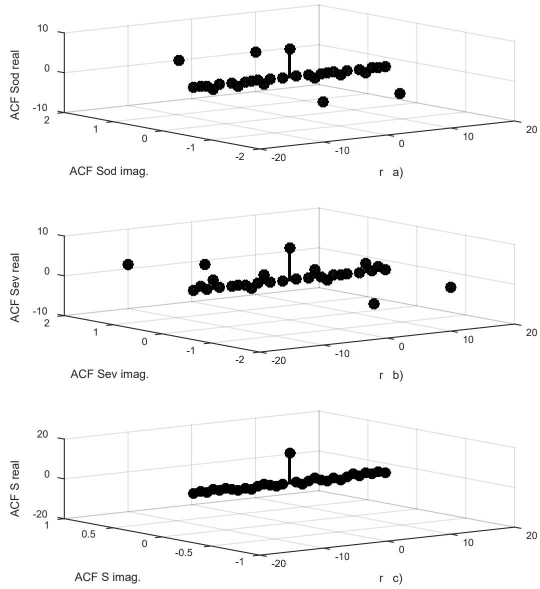

Fig. 1. ACFs of the GSS signal (25) (c) and its odd (a) and even (b) components.

In fact, this signal (accounting the possible equivalence transformations of PM signals) is the longest signal in the class of so-called complex Barker signals, which chips belong to the signal alphabet $\{-1, 1, i, -i\}$.

Another generalization of skew-symmetric sequences with odd lengths can be obtained, exploiting the following coding rule

$$\mu(L) = 0 \; or \; \pm i, \quad \mu(L+k) = (-1)^{k+1}\mu^*(L-k), \; k = 1, 2, \dots, L. \tag{26}$$

Here it should be pointed out that the coding rule (26) changes the roles of the even and odd parts of the digital signal (2). Namely, in contrast with the coding rule (15), when $N \equiv 1 mod 4$, $L \equiv 0 mod 2$, the reciprocal polynomials $\tilde{F}_{e\mu}(x)$ and $\tilde{F}_{o\mu}(x)$ have the following forms:

$$\tilde{F}_{e\mu}(x) = \sum_{k=0}^{L} \mu(L-2k)x^{2k} = F_{e\mu}^*(x), \tag{27}$$

$$\tilde{F}_{o\mu}(x) = \sum_{k=0}^{L} \mu(L-2k)x^{2k} = -F_{o\mu}^*(x). \tag{28}$$

Analogously, when $N \equiv 3 mod 4$, $L \equiv 1 mod 2$, the reciprocal polynomials $\tilde{F}_{e\mu}(x)$ and $\tilde{F}_{o\mu}(x)$ have the following forms:

$$\tilde{F}_{e\mu}(x) = \sum_{k=0}^{L} \mu(L-2k)x^{2k} = -F_{e\mu}^*(x), \tag{29}$$

$$\tilde{F}_{o\mu}(x) = \sum_{k=0}^{L} \mu(L-2k)x^{2k} = F_{o\mu}^*(x). \tag{30}$$

The reasonability of including of GSS signals, synthesized by the coding rule (26) with $\mu(L) = 0$, in the signal family, exploited by the communication devices, will be demonstrated by the means of the GSS signal with length $N = 33$:

$$\{\mu(k)\}_{k=0}^{32} = \{1,1,-1,-1,-1,-1,1,1,1,1,1,1, -1,1,1,1,0,1,-1,1,1,1,-1,1,-1,1,-1,-1, 1,-1,1,1,-1\}. \tag{31}$$

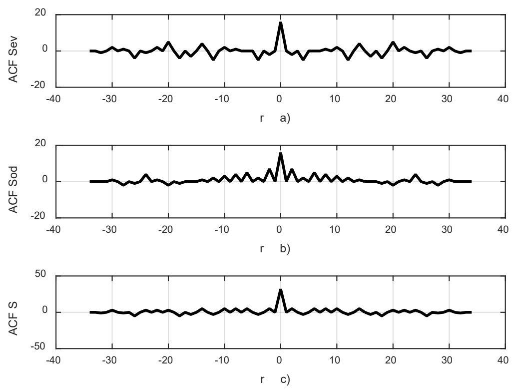

Fig. 2. ACFs of the GSS signal (31) (c) and its odd (a) and even (b) components.

Analogously, the reasonability of including of GSS signals, synthesized by the coding rule (24) with $\mu(L) = i$, in the signal family, exploited by the communication devices, will be demonstrated by the means of the GSS signal with length $N = 33$:

$$\{\mu(k)\}_{k=0}^{32} = \{1,-1,1,-1,1,1,-1,-1,1,-1, 1,-1,-1,1,1,1,i,1,-1,1,1,-1,-1,-1, -1,-1,1,1,-1,-1,-1,-1,-1\}. \tag{32}$$

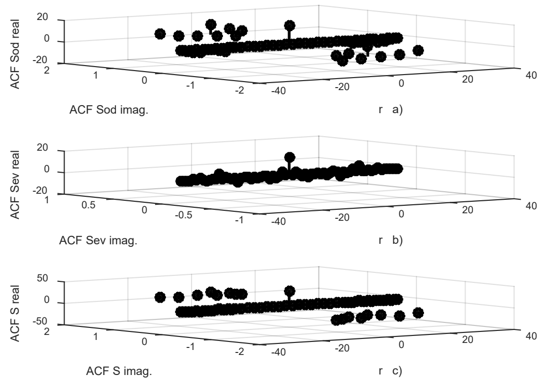

Fig. 3. ACFs of the GSS signal (32) (c) and its odd (a) and even (b) components.

The ACFs of GSS signals, presented on Fig. 1, Fig. 2 and Fig. 3 demonstrate that GSS signals possess high MFs and PSRs, which make them suitable for application in the radar sensor networks and remote control systems.

The practical importance the study, presented in the paper, ensues from the following circumstances.

First, at the one hand, the GSS signals, presented above, possess complex inner structure and can have great lengths, which allow their energy to be spread in very large spectral bands. As a result, the GSS signals can have both very small spectral density and energy, sufficient for normal operation of the radar sensor networks and remote control systems. At the other hand, according to the analyses in [12], [13] the exploitation of such signals provides both electromagnetic compatibility of a great amount of users, operating simultaneously, and high resistance in a hostile radio electronic environment. Moreover, the fact that the GSS signals, developed in the paper, form a large signal family allows the pseudo-random choosing of GSS signals from it to be managed by artificial intelligence and adaptive self-learning computer systems.

Second, the family of GSS signals can be extra extended in several ways. For example, according to the exploration, presented in our previous paper [11], it is possible GSS signals together with complementary pairs (CPs) of binary signals to be used in the recursive procedures for synthesis of quasi CPs (QCPs) [4], [5], which aggregated ACFs have very small relative quantity of non-zero sidelobes. Another way for extension of the GSS signal family is the application of transformations, preserving the autocorrelation properties of the PM signals (i.e. using all negative, reflective and alternate derivatives of the PM signals in the initial GSS signal family).

Third, it should be noted that according to the analysis in [14], every GSS signal can be implemented practically by the means of a single frequency channel, divided into two subchannels by quadrature phase manipulation (QPSK) or by two different types of polarization.

IV. Conclusions

In the paper two methods for synthesis of infinite families of PM signals with high MFs and PSRs are substantiated. The usage of the methods allows the electromagnetic compatibility of all working simultaneously electronic devices as well as their resistance in a hostile radio electronic environment to be improved. The study, conducted in the paper, could be useful during the modernization of the extant or development of new radar sensor networks and remote control systems.

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