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Mathematical model equation of the energy willow cutting unloading process from the slot hopper automated planter

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Abstract. **This work is a logical continuation of the authors' cycle of works devoted to construction of a mathematical model of the unloading process of cuttings from a slotted hopper. The purpose of the paper is a theoretical justification of the movement of an array of cuttings during gravity unloading. The article proposes consideration of the process of unloading the energy willow cuttings from the point of view of hydrodynamic multiphase systems. It is suggested to consider the set of cuttings as a pseudo-fluid consisting of two phases: discrete (cuttings) and continuous (air). Each of these phases can be considered as a continuous environment. Under such conditions, the task is reduced to consideration of the cuttings' discharge as the movement of a viscous non-contacting pseudo-liquid. Under such assumptions, the velocity field of the set of cuttings can be characterized by the Navier-Stokes equation, the solution of which will allow forming of an algorithm for building a mathematical model of the motion of such pseudo-fluid and moving to the calculation equations of the motion with initial and boundary conditions.**

Keywords: unloading of cuttings, energy willow, mathematical model, Navier–Stokes equation, Laplace transform, calculation equations.

I. INTRODUCTION

Recently, in the field of energy, more and more attention is paid to renewable energy sources. One of the most promising trends from the point of view of increasing volumes is biomass energy. Particularly, there is a trend towards increasing popularity of fuels from bioenergy crops, for which fast and productive fuels are needed. The most widespread in Ukraine, energy willow is propagated vegetatively by cuttings 20-25 cm long and 5-20 mm in diameter [1-3]. Today, the planting of such material is carried out by planters, in which the planting material is fed exclusively by hand, which significantly limits the possibilities of increasing the efficiency of the units. The theoretical study of the movement of cuttings during gravity unloading and the implementation of the obtained results in practice can help to create a planting machine [4-6].

In accordance with the scientific direction that is being developed at the Higher Educational Institution "Podillia State University", "Justification of the work process and parameters of the cuttings supply mechanism of the machine for planting energy willow" (state registration number 0119U100945), an automated system of supply and selection of planting material of woody energy crops is being developed.

Analysis of recent research and publications.

Many works have dealt with the issues of improving the unloading process of materials, but, despite the significant successes in this field, the dynamic processes of unloading loose homogeneous materials have not been

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studied enough. Thus, currently the main characteristics are considerably investigated and physical and mechanical properties of bulk materials, which to one degree or another affect the process of crypt formation, are considered. General directions of research in the field of uninterrupted functioning of hopper devices and the improvement of crypt breaking equipment for bulk cargoes with a wide range of physical and mechanical properties are reflected [7-17].

Numerous studies of the crypt formation process made it possible to establish only some dependencies that explain the essence of this process. The degree of influence of a huge number of different interrelated factors on crypt formation is difficult to assess practically and predict theoretically: it is a geometry of the hopper and outlet opening, and the physical and mechanical properties of materials, and the conditions of loading, storage, and release. Precisely, due to the difficulties in ensuring uniform continuous movement, which excludes the process of crypt formation, until now there is no universal feeding device that would work effectively with any loose material, and the variety of material that requires unloaded contributes to further searches for justifications movement of one or another material.

It is also difficult to overestimate the scientific and practical importance of studies of the mechanism of movement of loose materials under the influence of their own weight, since the physical and mechanical properties of these materials and the patterns of their termination have a decisive influence on the design of hoppers, as well as discharge devices and devices that stimulate discharge.

It should be emphasized that today there is no single theory of discharge of loose materials and crypt formation processes in the hopper.

The problem is even more complicated when it is necessary to ensure a uniform and continuous discharge of material in which one dimension (length) significantly exceeds the other two dimensions. Plant cuttings are an example of such material.

When creating an automatic planter for such material, the task of fast and accurate feeding of cuttings occurred, which led us to search for ways to justify the movement of cuttings during unloading from the storage tank [18-20].

Therefore, the study of this issue will continue to be relevant. The authors of this work made a significant contribution to the development of this issue in their previous works. One of the first steps in this direction was the construction of a mathematical model of the process of gravitational discharge of rod-like materials from slotted hoppers [21-23].

The authors also worked out general principles for building a mathematical model of the process of unloading cuttings from a hopper, defined boundary conditions and characteristics of their movement [23, 25].

The main assumptions about the nature of the movement of the stem-air mixture, which was presented in the form of a two-phase pseudo-liquid, were considered and substantiated. Thanks to this, some components of the equations could be neglected, and the existing equations of motion could be significantly simplified.

In previous studies, based on the assumptions made about the nature of the movement of a two-phase fluid, simplified equations were obtained [21,22]:

$$\frac{\partial \vec{u}}{\partial \bar{t}} = -\gamma \nabla \rho + \bar{\nu} \Delta \vec{u} + \Phi(\vec{u}_1 - \vec{u}) - \bar{g}\vec{e}_2 \tag{1}$$

$$\frac{\partial \vec{u}_1}{\partial \bar{t}} = -0.5 \frac{\delta}{1-\delta} \frac{\partial}{\partial \bar{t}} (\vec{u}_1 - \vec{u}) - \Phi_1 \int_0^{\bar{t}} \frac{\partial}{\partial \bar{t}} (\vec{u}_1 - \vec{u})(\bar{t} - \bar{\tau})^{-1/2} d\bar{\tau} - \Phi_2 (\vec{u}_1 - \vec{u}) - \bar{g}\vec{e}_2 \tag{2}$$

$$\operatorname{div} \vec{u} = 0, \quad \operatorname{div} \vec{u}_1 = 0 \tag{3}$$

Equations (1) - (3) became the basis for describing the process of unloaded cuttings from the hopper. It is necessary to add initial and boundary conditions to these equations, which in the new notation have taken the form:

Initial conditions:

$$\begin{aligned} \vec{u}\big|_{\bar{t}=0} &= \vec{u}_1\big|_{\bar{t}=0} = 0 \\ \rho\big|_{\bar{t}=0} &= 0 \end{aligned} \tag{4}$$

Boundary conditions:

$$\text{at } \bar{x}_2 = h(\bar{t})/L, \quad -\rho + \frac{2\mu V_0}{L}\frac{\partial u_2}{\partial \bar{x}_2} = 0 \tag{5}$$

where h - the thickness of the layer of cuttings

$$\frac{\partial u_1}{\partial \bar{x}_2} + \frac{\partial u_2}{\partial \bar{x}_{12}} = 0, \tag{6}$$

$$\dot{h} = V_0 T u_2, \tag{7}$$

a $\bar{x}_2 = -\operatorname{tg}\alpha(\bar{x}_1 + b/2L) \quad -\operatorname{ctg}\alpha h_0/L - b/2L < \bar{x}_1 < b/2L$

$$\sin\alpha u_1 + \cos\alpha u_2 = \frac{A\omega}{V_0}\sin 2\pi\bar{t}, \tag{8}$$

$$\cos 2\alpha\left(\frac{\partial u_1}{\partial \bar{x}_1} + \frac{\partial u_2}{\partial \bar{x}_2}\right) + 2\sin 2\alpha \frac{\partial u_1}{\partial \bar{x}_1} = \frac{gL\cos^2\alpha h(\bar{t})}{2\nu V_0}, \tag{9}$$

at $\bar{x}_2 = \operatorname{tg}\beta(\bar{x}_1 - b/2L) \quad b/2L < \bar{x}_1 < b/2L + \frac{\operatorname{ctg}\beta h_0}{L}$

$$\cos 2\beta\left(\frac{\partial u_1}{\partial \bar{x}_2} + \frac{\partial u_2}{\partial \bar{x}_1}\right) - 2\sin 2\beta \frac{\partial u_1}{\partial \bar{x}_1} = \frac{gL\cos^2\beta h(\bar{t})}{2\nu V_0}. \tag{10}$$

By applying the Laplace transform to determine the Fourier coefficients, a system of linear algebraic equations for the speed of movement of a pseudo-fluid was obtained (see (11), (12)) [21, 23]

$$U_1 = \bar{A}_{10} e^{-\sqrt{\lambda}\bar{x}_2} + \sum_{n=1}^{\infty} \bar{\lambda}_n e^{-\sqrt{\lambda_n}\bar{x}_2}\left(\bar{B}_{1n}\sin\frac{2\pi n}{M}\bar{x}_1 - \bar{B}_{2n}\cos\frac{2\pi n}{M}\bar{x}_1\right), \tag{11}$$

$$U_2 = -d/\lambda + \sum_{n=0}^{\infty} e^{-\sqrt{\lambda_n}\bar{x}_2}\left(\bar{B}_{1n}\cos\frac{2\pi n}{M}\bar{x}_1 + \bar{B}_{2n}\sin\frac{2\pi n}{M}\bar{x}_1\right), \tag{12}$$

where $A_{1n}, A_{2n}, B_{1n}, B_{2n}$ - the quantities that are unknown functions of the variable x_2.

$$\dot{B}_{1n} + \frac{2\pi n}{M} A_{2n} = 0, \quad \dot{B}_{2n} - \frac{2\pi n}{M} A_{1n} = 0.$$

$$\overline{A}_{1n} = -\sqrt{\lambda_n}\frac{M}{2\pi n}\overline{B}_{2n}, \quad \overline{A}_{2n} = \sqrt{\lambda_n}\frac{M}{2\pi n}\overline{B}_{1n}.$$

$$\overline{\lambda}_n = \sqrt{\lambda\left(\frac{M}{2\pi n}\right)^2 + 1} \tag{13}$$

Formulas (11), (12) provide a general solution to the system of equations of fluid motion. To find values $\overline{B}_{1n}, \overline{B}_{2n}$ it is necessary to use the boundary conditions (5) - (10).

This work is the final part of a cycle of works devoted to the construction of a mathematical model of the process of unloading cuttings from a slotted hopper, therefore its purpose is to derive calculation formulas for the movement of an array of cuttings during their gravitational unloading from a slotted hopper.

II. Material and Methods

The theoretical basis of the research was the work of domestic and foreign scientists, in which scientific methods were developed to justify the process of unloading bulk material from containers, with the development of issues of solving the problems of crypt formation and continuous unloading of material. Based on the analysis of existing solutions for the movement of material during gravity dumping, a model of the movement of cylindrical bodies (cuttings) during free unloading from the hopper was created.

For preliminary studies, the hopper model (Fig. 1) was used as a basis, in which consideration of the process is limited to a two-dimensional model (in the x_1x_2 plane), since it is believed that the movement of cuttings in the hopper does not depend on the x_3 coordinate, due to the presence of walls parallel to the x_1x_2 plane, which limit the movement of cuttings along the x_3 axis.

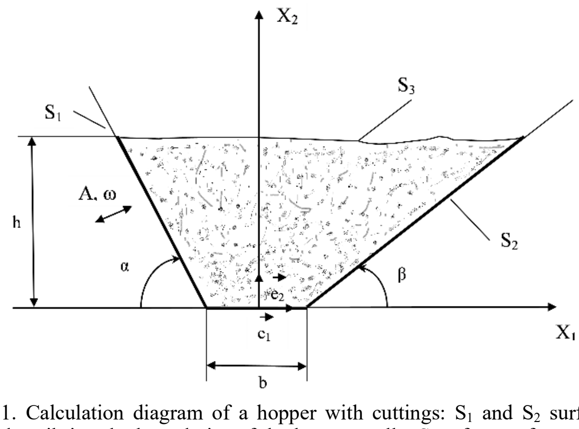

Fig. 1. Calculation diagram of a hopper with cuttings: S_1 and S_2 surfaces describ-ing the boundaries of the hopper walls; S_3 – free surface of the hopper boundaries; α and β - angles to the horizontal plane of the two half-planes describing the bunker model; h - thickness of the layer of cuttings; b –width of the unloading window; A, ω - amplitude and circular speed of harmonic oscillations parallel to the axis x_2; e_1, e_2, e_3 – unit vectors of the Cartesian coordinate system.

At the same time, based on the analysis of existing solutions, a number of assumptions were made, which allowed to consider the gravitationally unloaded cuttings from the point of view of hydrodynamic multiphase systems.

According to this approach, the set of cuttings is considered as a fluid consisting of two phases: a discrete phase formed by the cuttings and a continuous phase - a gaseous medium (air). Each of these phases is considered as a continuous environment, which allowed us to consider the discharge as the movement of a viscous incompressible fluid. The velocity field of such a pseudo-fluid must satisfy the Navier-Stokes equation.

III. Results and Discussion

As noted in the authors' previous studies the system of linear algebraic equations was obtained for determination of the Fourier coefficients of the Laplace transformation of the speed of movement of the pseudo-fluid (see (11), (12)) [26,27].

In the future, we will limit ourselves to the case of a symmetrical hopper in order to obtain calculation formulas for the velocity of fluid movement. Within the framework of the adopted model of the hopper, this means that for the angles α and β the equality α= β is fulfilled. This restriction, on one hand, does not deny the commonality of the previously presented results, and on the other hand, it allows obtaining a solution to problem (1) - (3), (4) - (10) in closed analytical form.

So, let α=β, then the system of linear algebraic equations takes the form:

$$\overline{B}_0 \frac{1-e^{-\sqrt{x}\,\overline{h}_0}}{\sqrt{\lambda}}\cos\alpha + \overline{B}_1\left(\cos\alpha\Phi_c - \overline{\lambda}_1\sin\alpha\Phi_s\right) - \overline{B}_2\left(\overline{\lambda}_1\sin\alpha\Phi_c + \cos\alpha\Phi_s\right) = \overline{h}_0\,\mathrm{ctg}\,\alpha\left(\overline{N}_3 - \overline{d}\right) \tag{14}$$

$$\cos 2\alpha \frac{1-e^{-\sqrt{\lambda}\,\overline{h}_0}}{\mathrm{tg}\,\alpha}\overline{B}_0 - \overline{B}_1 G_c + \overline{B}_2 G_s = -f\,\overline{N}\,\overline{h}_0\,\mathrm{ctg}\,\alpha \tag{15}$$

$$\cos 2\alpha \frac{1-e^{-\sqrt{\lambda}\,\overline{h}_0}}{\mathrm{tg}\,\alpha}\overline{B}_0 + \overline{B}_1 G_c + \overline{B}_2 G_s = -f\,\overline{N}\,\overline{h}_0\,\mathrm{ctg}\,\alpha \tag{16}$$

Here

$$\overline{N} = \frac{ga\cos^2\alpha\overline{h}}{vA\omega} \tag{17}$$

$$\Phi_s = \frac{1}{\lambda_1 tg^2\alpha + \frac{4\pi^2}{M^2}}\left[\sqrt{\lambda_1}tg\alpha\sin\frac{\pi\overline{b}}{M} + \frac{2\pi}{M}\left(\cos\frac{\pi\overline{b}}{M} + e^{-\sqrt{\lambda_1}\overline{h}_0}\right)\right] \tag{18}$$

$$\Phi_c = \frac{1}{\lambda_1 tg^2\alpha + \frac{4\pi^2}{M^2}}\left[\sqrt{\lambda_1}tg\alpha\left(\cos\frac{\pi\overline{b}}{M} + e^{-\sqrt{\lambda_1}\overline{h}_0}\right) - \frac{2\pi}{M}\sin\frac{\pi\overline{b}}{M}\right], \tag{19}$$

$$G_c = \frac{2\pi}{M}\left(1 + \frac{\lambda_1 M^2}{4\pi^2}\right)\cos 2\alpha\,\Phi_s + 2\sqrt{\lambda_1}\sin 2\alpha\,\Phi_c \tag{20}$$

$$G_s = 2\sqrt{\lambda_1}\sin 2\alpha\,\Phi_s - \frac{2\pi}{M}\left(1+\frac{\lambda_1 M^2}{4\pi^2}\right)\cos 2\alpha\,\Phi_c \tag{21}$$

Let's get the solution of this system of equations. It follows from (15) and (16).

$$\overline{B}_1 = 0 \tag{22}$$

$$\overline{B}_0 = -\frac{b_2 G_s \operatorname{tg}\alpha + f\overline{N}\overline{h}_0}{\cos 2\alpha\left(1-e^{-\sqrt{\lambda}h_0}\right)} \tag{23}$$

Substituting (22) and (23) into (14), finally have:

$$\overline{B}_0 = \frac{\sqrt{\lambda}\overline{h}_0 \operatorname{ctg}\alpha}{1-e^{-\sqrt{\lambda}h_0}}\begin{bmatrix}\overline{N}_3 - \overline{d} - \\ -\dfrac{\left(\overline{\lambda}_1 \sin\alpha\,\Phi_c + \cos\alpha\Phi_s\right)\left(\sqrt{\lambda\cos 2\alpha}\operatorname{ctg}\alpha\left(\overline{N}_3-\overline{d}\right)+f\overline{N}\right)}{G_s+\sqrt{\lambda}\operatorname{ctg}\alpha\cos 2\alpha\left(\overline{\lambda}_1\sin\alpha\Phi_c+\cos\alpha\Phi_s\right)}\end{bmatrix} \tag{24}$$

$$\overline{B}_2 = -\frac{\overline{h}_0 \operatorname{ctg}\alpha\left(\sqrt{\lambda}\cos 2\alpha\operatorname{ctg}\alpha\left(\overline{N}_3-\overline{d}\right)+f\overline{N}\right)}{G_s+\sqrt{\lambda}\operatorname{ctg}\alpha\cos 2\alpha\left(\overline{\lambda}_1\sin\alpha\Phi_c+\cos\alpha\Phi_s\right)}. \tag{25}$$

Formulas (22), (24), (25) provide a solution to the system of equations (14) - (16).

Next, using (22), (24) and (25), we obtain the following formulas for the Laplace transform U_1 and U_2 of the speed of movement of a pseudo fluid:

$$U_1 = \overline{h}_0 ctg\alpha\begin{bmatrix}\dfrac{\sqrt{\lambda}}{1-e^{-\sqrt{\lambda}\overline{x}_2}}\left(\dfrac{2\pi}{q^2+4\pi^2}+\dfrac{d}{\lambda}-D_1D_2\right)e^{-\sqrt{\lambda}\overline{x}_2}+ \\ +D_2\dfrac{\sqrt{\lambda_1}M}{2\pi}e^{-\sqrt{\lambda_1}\overline{x}_2}\cos\dfrac{2\pi}{M}\overline{x}_1\end{bmatrix} \tag{26}$$

$$U_2 - \frac{d}{\lambda} - \overline{h}_0 ctg\alpha\, D_2 e^{-\sqrt{\lambda_1}\overline{x}_2}\sin\frac{2\pi}{M}\overline{x}_1 \tag{27}$$

$$D_1 = \sqrt{\lambda_1}\frac{M}{2\pi}\sin\alpha\,\Phi_c + \cos\alpha\,\Phi_s \tag{28}$$

$$D_2 = \frac{\sqrt{\lambda}\cos 2\alpha\, ctg\alpha\left(\dfrac{2\pi}{q^2+4\pi^2}+\dfrac{d}{\lambda}\right)+f\overline{N}}{G_s+\sqrt{\lambda}ctg\alpha\cos 2\alpha\, D_1}. \tag{29}$$

These formulas can be simplified, taking into account the fact that the value $\frac{2\pi}{M} << 1$, if $\alpha \neq \frac{\pi}{2}$ and $\overline{h}_0 = \frac{h_0}{2\alpha} >> 1$.

In this approximation, we have:

$$D_1 \approx \sqrt{\lambda}\frac{M}{2\pi}\sin\alpha\,\Phi_c \tag{30}$$

$$G_s \approx -\frac{\lambda M}{2\pi}\cos 2\alpha\,\Phi_c \tag{31}$$

$$\Phi_c \approx \frac{ctg\alpha}{\sqrt{\lambda}}\cos\frac{\pi\overline{b}}{M} \tag{32}$$

Substitute (30)–(32) into (29) and (26), (27) and, after performing the necessary transformations, we have:

$$U_2 \approx -\frac{d}{\lambda}+\frac{2\pi\overline{h}_0}{M(1-\cos\alpha)\cos\frac{\pi\overline{b}}{M}}\left(\operatorname{ctg}\alpha D+\frac{f\overline{N}}{\sqrt{\lambda_1}\cos 2\alpha}\right)e^{-\sqrt{\lambda_1}\overline{x}_2}\sin\frac{2\pi}{M}\overline{x}_1 \tag{33}$$

$$U_1 \approx \overline{h}_0 ctg\alpha\begin{bmatrix}\dfrac{\sqrt{\lambda}}{1-e^{-\sqrt{\lambda}\overline{h}_0}}\left(D\left(1+ctg\dfrac{\alpha}{2}ctg\alpha\right)+\dfrac{f\overline{N}ctg\dfrac{\alpha}{2}}{\sqrt{\lambda_1\cos 2\alpha}}\right)e^{-\sqrt{\lambda}\overline{x}_2} - \\ -\dfrac{\sqrt{\lambda_1}}{(1-\cos\alpha)\cos\dfrac{\pi\overline{b}}{M}}\left(D+\dfrac{f\overline{N}}{\sqrt{\lambda_1\cos 2\alpha}ctg\alpha}\right)e^{-\sqrt{\lambda_1}\overline{x}_2}\cos\dfrac{2\pi}{M}\overline{x}_1\end{bmatrix}. \tag{34}$$

$$D = \frac{2\pi}{q^2+4\pi^2}+\frac{d}{\lambda}, \qquad \overline{N} = \frac{ga\cos^2\alpha\,\overline{h}}{vA\omega}.$$

Formulas (33), (34) give an approximate value of the functions U_1 and U_2.

In order to use (33), (34) to solve the original problem, it is sufficient to apply the inverse transformation to the Laplace transform [26]

$$u_1 = \frac{1}{2\pi i}\int_{\overline{a}-i\infty}^{\overline{a}+i\infty}U_1 e^{q\overline{t}}dq, \quad u_2 = \frac{1}{2\pi i}\int_{\overline{a}-i\infty}^{\overline{a}+i\infty}U_2 e^{q\overline{t}}dq \tag{35}$$

$\overline{a} > 0$.

The next step is to calculate the integrals (35). For this purpose, we examine the functions U_1 and U_2 as a function - a parameter of the Laplace transform. As follows from (33) and (34), this function depends on $\sqrt{q}$. Therefore, bearing in mind that q can take a complex value, one of the branches of the analytical function $\sqrt{q}$ should be singled out. To do this, in the complex plane of the variable q, we will make a cut along the negative real semiaxis ($\operatorname{Re}q < 0$, $\lim q < 0$). In such a complex plane, we select a branch $\sqrt{q}$ for which $\operatorname{Re}\sqrt{q} \geq 0$ and $-\pi < \arg q \leq \pi$. It is easy to see that the functions U_1 and U_2 are analytic functions of the complex variable q excluding the above cut and the points $q = 0$, $q = \pm i2\pi$. Moreover, the points $q = \pm i2\pi$ are special points of the pole type, and the point $q = 0$ is a branching point of the algebraic type. In addition, the functions U_1 and U_2 tend to zero. Such properties of the functions U_1 and U_2 guarantee the applicability of the remainder theorem [26] and allow the calculation of the integral (35) to be replaced by the calculation of the remainders at points $q = \pm i2\pi$ and integrals of these functions over a circle with an infinitesimally small radius and centered at the branching point $q = 0$. Residuals can be calculated using the formula [25, 26].

$$resU_{1,2}(\pm i2\pi) = \lim_{q\to\pm i2\pi}(q \pm i2\pi)U_{1,2}(q) \tag{36}$$

Thus, based on the above, we get

$$u_1(x_1,x_2,t) = res\left(U_1(2\pi i)e^{i\omega t}\right)+res\left(U_1(-2\pi i)e^{-i\omega t}\right)-\lim_{\varepsilon\to 0}\int_{C_\varepsilon}U_1 e^{\frac{qt\omega}{2\pi}}dq \tag{37}$$

$$u_2(x_1,x_2,t) = res\left(U_2(2\pi i)e^{i\omega t}\right)+res\left(U_2(-2\pi i)e^{-i\omega t}\right)-\lim_{\varepsilon\to 0}\int_{C_\varepsilon}U_2 e^{\frac{qt\omega}{2\pi}}dq \tag{38}$$

Here - denotes an excess, - a circle of radius ε centered at a point $q = 0$.

Using formula (36), we get

$$u_{11} = res\left(U_1(2\pi i)e^{i\omega t}\right) + res\left(U_1(-2\pi i)e^{-i\omega t}\right) = \frac{\sqrt{\frac{\omega}{2\nu}}h_0 \operatorname{ctg}\alpha\left(1 + ctg\alpha ctg\frac{\alpha}{2}\right)\left[\cos\gamma - \sin\gamma - e^{-\gamma}\left(\cos(\gamma - \bar{\gamma}) - \sin(\gamma - \bar{\gamma})\right)\right]}{1 + e^{-2\gamma} - 2\cos\bar{\gamma}e^{-\bar{\gamma}}} e^{-\bar{\gamma}\frac{x_2}{h_0}} - \frac{\sqrt{\frac{\omega}{2\nu}}h_0 \operatorname{ctg}\alpha(\cos\gamma - \sin\gamma)}{(1 - \cos\alpha)\cos\frac{\pi b}{\overline{\overline{M}}}} e^{-\bar{\gamma}\frac{x_2}{h_0}} \cos\frac{2\pi x_1}{M} \tag{39}$$

$$u_{21} = res\left(U_2(2\pi i)e^{i\omega t}\right) + res\left(U_2(-2\pi i)e^{-i\omega t}\right) = \frac{2\pi h_0 \operatorname{ctg}\alpha\cos\gamma}{(b + 2h_0 \operatorname{ctg}\alpha)(1 - \cos\alpha)\cos\frac{\pi b}{\overline{\overline{M}}}} e^{-\bar{\gamma}\frac{x_2}{h_0}} \sin\frac{2\pi x_1}{M} \tag{40}$$

In formulas (39), (40), dimensionless variables $\bar{t}, \bar{x}_1, \bar{x}_2$ are replaced by dimensional t, x_1, x_2 variables. The value $\overline{M} = b + h_0 \operatorname{ctg}\alpha$, b is the width of the unloaded window,

h_0 - the distance from the free border of the layer of cuttings to the plane of the unloaded window at the moment of time $t=0$.

Value $\gamma = \omega t - \sqrt{\frac{\omega}{2\nu}}x_2, \bar{\gamma} = \sqrt{\frac{\omega}{2\nu}}h_0$

Let's calculate the integrals in (37), (38). For this, we will assume that the distance from the border of the free surface of the cuttings to the plane of the unloading window changes over time according to a linear law

$$h(t) = ct + h_0 \tag{41}$$

where c – is some constant to be determined.
Then the Laplace transform of this function

$$\bar{h}(q) = \frac{c}{\bar{q}^2} + \frac{h_0}{q} \tag{42}$$

Substitute (42) into (36) and (33) into (34). Having made the necessary transformations, we have

$$u_{12} = \lim_{\varepsilon \to 0} \int_{c_\varepsilon} U_1 e^{\frac{qt\omega}{2\pi}} dq = \frac{\operatorname{ctg}\alpha\, g}{A\omega}\left[\left(1 + \operatorname{ctg}\alpha\operatorname{ctg}\frac{\alpha}{2}\right)\frac{2\bar{\rho}_1(1-\delta)}{\omega\bar{\rho}\delta} - \frac{\overline{M}f \operatorname{ctg}\frac{\alpha}{2}\sin 2\alpha}{4\pi\cos^2\alpha\nu} \times (\bar{c} + h_0)e^{-\frac{2\pi}{M}x_2} + \left(\frac{4\pi h_0\bar{\rho}_1(1-\delta)}{(1-\cos\alpha)\overline{M}\cos\frac{\pi b}{\overline{\overline{M}}}\bar{\rho}\omega\delta} + \frac{h_0 \operatorname{tg}2\alpha f}{4(1-\cos\alpha)\nu}(\overline{C} + h_0)\right) e^{-\frac{2\pi}{\overline{\overline{M}}}x_2} \cos\frac{2\pi}{\overline{\overline{M}}}x_1\right] \tag{43}$$

$$u_{22} = \lim_{\varepsilon \to 0} \int_{c_\varepsilon} U_2 e^{\frac{qt\omega}{2\pi}} dq = \frac{2g\bar{\rho}_1(1-\delta)}{A\omega\bar{\rho}\delta}\left(1 - \frac{2\pi h_0 \operatorname{ctg}\alpha}{(1-\cos\alpha)\overline{M}\cos\frac{\pi b}{\overline{\overline{M}}}} e^{-\frac{2\pi}{\overline{\overline{M}}}x_2} \sin\frac{2\pi}{\overline{\overline{M}}}x_1\right) - \frac{h_0 fg\cos^2\alpha}{(1-\cos\alpha)\cos\frac{\pi b}{\overline{\overline{M}}}\cos 2\alpha\nu A\omega}(\overline{C} + h_0)e^{-\frac{2\pi}{\overline{\overline{M}}}x_2} \sin\frac{2\pi}{\overline{\overline{M}}}x_1, \tag{44}$$

$$\overline{C} = 2ca/\pi .$$

Finally, we have the following calculation formulas for the components of the speed of movement of the discrete phase (set of cuttings) of the fluid

$$u_1 = u_{11} - u_{12} \tag{45}$$

$$u_2 = u_{21} - u_{22} \tag{46}$$

where values u_{11}, u_{12}, u_{21}, u_{22} are determined by formulas (39), (40) and (43), (44).

As follows from (43), (44), the values u_{12} and u_{22} do not depend on the temporal variable t but depend on the spatial variables x_1 and x_2. The values u_{11} and u_{21} depend on the time variable according to the harmonic law in accordance with the vibrational oscillations that affect one of the walls of the hopper. In addition, these values depend on the physico-mechanical and geometric parameters of the cuttings (average density, the radius of the circle in area coincides with the cross-sectional area of the cuttings, the coefficient of dry friction between the cuttings), the parameters of the hopper (the angle of inclination of the walls, the width of the unloading window, the coefficient of friction around the walls of the hopper), amplitude and frequency of harmonic vibrations, kinematic coefficient of viscosity and air density.

In addition to the specified parameters, the speed components u_1 and u_2 implicitly depend on the function $h(t)$ (see 41). As follows from the boundary condition, this function must satisfy a nonlinear differential equation of the first order

$$\dot{h} = A\omega u_2(x_1, h(t), t) \tag{47}$$

The solution of this equation, in the general case, can be found only by numerical methods with the help of a computer. However, if we assume that the function $h(t)$ depends on time according to a linear law (see 41)

$$h(t) = ct + h_0,$$

Then you can define a constant c.

Indeed, let's put in (47) $x_1 = 0$. Then, taking into account (40) and (44), we have

$$c = -\frac{2g\vec{p}_1(1-\delta)}{\omega\vec{p}\delta} \tag{48}$$

This constant should be substituted in (39), (40) and (43), (44). This completes the construction of a mathematical model of the process of unloaded cuttings from the hopper.

IV. CONCLUSIONS

To date, planters of woody energy crops are known exclusively with manual laying of planting material. Therefore, development of automation systems for this process will contribute to the possibility of rapid expansion of areas under energy plantations.

The simplest way to move the material when it is unloaded, is its movement under the influence of gravitational forces. The theoretical foundations of such a movement do not have a single approach, and the specifics of the material for planting energy willow create additional difficulties for the development of a mathematical model of this process. Building a mathematical model of the movement of energy willow cuttings will allow automating the planting process.

Accepting several assumptions, it is proposed to consider gravitationally unloaded cuttings from the point of view of hydrodynamic multiphase systems. According to this approach, the collection of cuttings is considered as an incompressible fluid consisting of two phases: discrete, formed by the cuttings, and a continuous phase (gaseous - the medium between the cuttings). And by applying the Laplace transformation to determine the Fourier coefficients, the system of linear algebraic equations of the motion speed of the pseudo-fluid is obtained (see (12), (13)), which give a general solution to the system of equations of motion of such a pseudo-fluid with the outline of initial and boundary conditions.

In order to obtain calculation formulas for the speed of fluid movement, we limited ourselves to the case of a symmetric hopper, which allowed us to obtain a solution to problem (1) - (3), (4) - (10) in a closed analytical form.

Formulas (33), (34) give an approximate value of the functions U_1 and U_2 with a relative error of less than 5%, which is sufficient for practical calculations. In order to obtain the solution of the original problem with the help of (33), (34), it is sufficient to apply the inverse transformation to the Laplace transform [26, 27].

Thus, the calculation formulas for the components of the speed of movement of the discrete phase (set of cuttings) of the fluid (45) and (46) were finally obtained, the individual components of which u_{11}, u_{12}, u_{21}, u_{22} are determined by formulas (39), (40) and (43), (44).

This mathematical model describes the movement of cuttings of energy tree crops when unloading a slotted hopper, which creates opportunities for designing mechanisms for automatic unloading of such material, which in turn will allow to propose the design of automatic planters.

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