# Synthesis of Eight Middle Lost Mechanisms Finally Discharged Discreet Positions 

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#### Abstract

The author has developed the algorithm for the synthesis of eight kinematic structures to generate random function of the positions by given n-number approximate nodes. Synthesis was performed at extreme distances at $\mathbf{n} \leq 6$. In practice, an approximate synthesis is proposed, with the error being zero only in the approximate nodes, and in the remaining points the error is plotted graphically. According to preset 6 approximate nodes we determine the metric parameters of a family of mechanisms from which we choose the optimal (with the least relative error in the movement of the executive unit). The determination of the position, speed and acceleration function of the executive unit is shown graphically. To establish the authenticity of synthesis is an example.


Keywords: eight axial lever mechanisms, metric syntheses at far-off positions, marginal synthesis, function of the position.

## I. Introduction

When it is necessary to determine the geometric dimensions of an arbitrary mechanism that generates an arbitrary continuous function of positions and this generated function has $n$ number of predetermined points called extremely distant points [Galabov 1992], precise points [Galabov 1992], approximation nodes [Enchev 1986] or points corresponding to $n$ discrete positions of the mechanism, a metric synthesis of this mechanism must be performed. We will call the specified metric synthesis from [Marinov 2014] marginal (finite, at finitely distant precise points) including $n$ number of discrete positions, fixed on the continuous function of the positions of the investigated mechanisms.

The metric synthesis of this mechanism in extremely distant positions can be performed by means of the vectormatrix method, using the generalized approach applied to planar lever mechanisms [Marinov 2012].


Fig. 1 shows an eight-link lever mechanism serving a transport manipulator.

## II. MATERIALS AND METHODS

## A. Synthesis of the mechanism:

The synthesis conditions are determined by the relative discrete angular orientations $\boldsymbol{\theta}_{\mathbf{1 i}}$ of link 1 and the relative linear displacements $\mathbf{S}_{\mathbf{1 i}}$ of link 3. The coordinates of the center of the spherical pair B, at any discrete position of the mechanism, are:

$$
\begin{align*}
& X_{B_{i}}=X_{B_{1}}+S_{1 i} \\
& Y_{B_{i}}=Y_{B_{1}}  \tag{1}\\
& Z_{B_{i}}=Z_{B_{1}}
\end{align*}
$$

After a kinematic inversion of the mechanism, at stand unit $\mathbf{1}$, the coordinates of any inverted position of
the center of the spherical pair $\mathbf{B}$ have the following form:

$$
\left[\begin{array}{c}
\mathrm{X}_{\mathrm{Bi}}^{\prime}  \tag{2}\\
\mathrm{Y}_{\mathrm{Bi}}^{\prime} \\
\mathrm{Z}_{\mathrm{Bi}}^{\prime}
\end{array}\right]=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \theta_{\mathrm{li}} & \sin \theta_{\mathrm{li}} \\
0 & -\sin \theta_{\mathrm{ti}} & \cos \theta_{\mathrm{li}}
\end{array}\right]\left[\begin{array}{c}
\mathrm{X}_{\mathrm{B}_{1}}+\mathrm{S}_{\mathrm{ii}} \\
\mathrm{Y}_{\mathrm{B}_{1}} \\
\mathrm{Z}_{\mathrm{B}_{1}}
\end{array}\right]
$$

The geometric constraint condition of unit 2 can be written in vector form as:

$$
\begin{equation*}
\left[\overrightarrow{\mathrm{RB}_{\mathrm{i}}^{\prime}}-\overrightarrow{\mathrm{RA}_{1}}\right]^{\mathrm{T}}\left[\overrightarrow{\mathrm{RB}_{\mathrm{i}}^{\prime}}-\overrightarrow{\mathrm{RA}_{1}}\right]=\left[\overrightarrow{\mathrm{RB}_{1}}-\overrightarrow{\mathrm{RA}_{1}}\right]^{\mathrm{T}}\left[\overrightarrow{\mathrm{RB}_{1}}-\overrightarrow{\mathrm{RA}_{1}}\right] \tag{3}
\end{equation*}
$$

In scalar-matrix form, condition (3) has the following form:

$$
\left[\begin{array}{c}
\mathrm{X}_{\mathrm{B}^{\prime} \mathrm{i}}  \tag{4}\\
\mathrm{Y}_{\mathrm{B}^{\prime} \mathrm{i}}-\mathrm{Y}_{\mathrm{A} 1} \\
\mathrm{Z}_{\mathrm{B}^{\prime} \mathrm{i}}-\mathrm{Z}_{\mathrm{A} 1}
\end{array}\right]^{\mathrm{T}}\left[\begin{array}{c}
\mathrm{X}_{\mathrm{B}^{\prime} \mathrm{i}} \\
\mathrm{Y}_{\mathrm{B}^{\prime} \mathrm{i}}-\mathrm{Y}_{\mathrm{A} 1} \\
\mathrm{Z}_{\mathrm{B}^{\prime} \mathrm{i}}-\mathrm{Z}_{\mathrm{A} 1}
\end{array}\right]=\left[\begin{array}{c}
\mathrm{X}_{\mathrm{B} 1} \\
\mathrm{Y}_{\mathrm{Bl}}-\mathrm{Y}_{\mathrm{A} 1} \\
\mathrm{Z}_{\mathrm{B} 1}-\mathrm{Z}_{\mathrm{A} 1}
\end{array}\right]^{\mathrm{T}}\left[\begin{array}{c}
\mathrm{X}_{\mathrm{B} 1} \\
\mathrm{Y}_{\mathrm{BI}}-\mathrm{Y}_{\mathrm{A} 1} \\
\mathrm{Z}_{\mathrm{B} 1}-\mathrm{Z}_{\mathrm{Al}}
\end{array}\right]
$$

At $\mathbf{n}$ extremely distant positions of the mechanism, $\mathbf{n}$ 1 equations of the type (4) can be written, namely:

$$
\begin{equation*}
\mathrm{a}_{\mathrm{i}} \mathrm{X}_{\mathrm{B}_{1}}+\mathrm{b}_{\mathrm{i}} \mathrm{Y}_{\mathrm{A}_{1}}+\mathrm{c}_{\mathrm{i}} \mathrm{Z}_{\mathrm{A}_{1}}=\mathrm{d}_{\mathrm{i}} \tag{5}
\end{equation*}
$$

where

$$
\begin{align*}
& \mathrm{a}_{\mathrm{i}}=2 \mathrm{~S}_{1 \mathrm{i}}, \\
& \mathrm{~b}_{\mathrm{i}}=-2\left(\mathrm{Y}_{\mathrm{B}^{\prime} \mathrm{i}}-\mathrm{Y}_{\mathrm{Bl}}\right),  \tag{6}\\
& \mathrm{c}_{\mathrm{i}}=-2\left(\mathrm{Z}_{\mathrm{B}^{\prime} \mathrm{i}}-\mathrm{Z}_{\mathrm{Bl}}\right), \\
& \mathrm{d}_{\mathrm{i}}=\mathrm{Y}_{\mathrm{Bl}}^{2}+\mathrm{Z}_{\mathrm{Bl}}^{2}-\mathrm{Y}_{\mathrm{B}^{\prime} \mathrm{i}}^{2}-\mathrm{Z}_{\mathrm{B}^{\prime} \mathrm{i}}^{2}-\mathrm{S}_{\mathrm{li}}^{2}, \mathrm{i}=2,3, \ldots, \mathrm{n}
\end{align*}
$$

When $\mathbf{n}=\mathbf{4}$, there are three equations of the type (5). Two of the parameters of the mechanism ( $\mathbf{Y}_{\mathbf{B} 1}$ and $\mathbf{Z}_{\mathbf{B} 1}$ ) are assigned constant values, the system of equations (5) is solved and the remaining three parameters $\left(\mathbf{X}_{\mathbf{B} 1}, \mathbf{Y}_{\mathbf{A} \mathbf{1}}\right.$ and $\mathbf{Z}_{\mathbf{A} 1}$ ) are determined.

With $\mathbf{n}=\mathbf{5}$, there are four equations of the type (5). In this case, the synthesis can be done in two ways. In both ways, three $\left(\mathbf{X}_{\mathbf{B 1} 1}, \mathbf{Y}_{\mathbf{A 1}}\right.$ and $\left.\mathbf{Z}_{\mathbf{A} \mathbf{1}}\right)$ are assumed for calculation parameters.

In the first way, one of the remaining two parameters $\left(\mathbf{Z}_{\mathbf{B} 1}\right)$ is assigned a constant value, and the other $\left(\mathbf{Y}_{\mathbf{B} 1}\right)$ is assumed to be variable within certain limits and with a certain step.

The calculation parameters are determined from the first three equations of (5):

$$
\begin{align*}
& \mathrm{X}_{\mathrm{B}_{1}}=\mathrm{X}_{\mathrm{B}_{1}}\left(\mathrm{Y}_{\mathrm{B}_{1}}\right), \\
& \mathrm{Y}_{\mathrm{A}_{1}}=\mathrm{Y}_{\mathrm{A}_{1}}\left(\mathrm{Y}_{\mathrm{B}_{1}}\right),  \tag{7}\\
& \mathrm{Z}_{\mathrm{A}_{1}}=\mathrm{Z}_{\mathrm{A}_{1}}\left(\mathrm{Y}_{\mathrm{B}_{1}}\right) .
\end{align*}
$$

With the fourth equation, the function of the variable parameter is written:

$$
\begin{equation*}
\mathrm{F}\left(\mathrm{Y}_{\mathrm{BI}}\right)=\mathrm{a}_{5} \cdot \mathrm{X}_{\mathrm{B}_{1}}+\mathrm{b}_{5} \cdot \mathrm{Y}_{\mathrm{A}_{1}}+\mathrm{c}_{5} \cdot \mathrm{Z}_{\mathrm{A}_{1}}-\mathrm{d}_{5} . \tag{8}
\end{equation*}
$$

Graphs of functions (7) and (8) are drawn. From the graph of the function (8), the values of the variable parameter $\mathbf{Y}_{\mathbf{B 1}}$ are determined at the points where this function is zeroed and for which the system of equations (5) has a solution. For the thus reported values of the variable parameter from the graphs of the functions (7), the calculation parameters $\left(\mathbf{X}_{\mathbf{B 1}}, \mathbf{Y}_{\mathbf{A} \mathbf{1}}\right.$ and $\left.\mathbf{Z}_{\mathbf{A} 1}\right)$ are reported.

In the second way, the synthesis can be carried out by assuming the two parameters ( $\mathbf{Y}_{\mathbf{B 1}}$ and $\mathbf{Z}_{\mathbf{B 1}}$ ) to be variable within certain limits and at certain steps of changes.

The calculation parameters are determined from the first three equations of (5):

$$
\begin{align*}
& \mathrm{X}_{\mathrm{B}_{1}}=\mathrm{X}_{\mathrm{B}_{1}}\left(\mathrm{Y}_{\mathrm{B}_{1}}, \mathrm{Z}_{\mathrm{B}_{1}}\right), \\
& \mathrm{Y}_{\mathrm{A}_{1}}=\mathrm{Y}_{\mathrm{A}_{1}}\left(\mathrm{Y}_{\mathrm{B}_{1}}, \mathrm{Z}_{\mathrm{B}_{1}}\right),  \tag{9}\\
& \mathrm{Z}_{\mathrm{A}_{1}}=\mathrm{Z}_{\mathrm{A}_{1}}\left(\mathrm{Y}_{\mathrm{B}_{1}}, \mathrm{Z}_{\mathrm{B}_{1}}\right) .
\end{align*}
$$

With the fourth equation, a function of the variable parameters is written:

$$
\begin{equation*}
\mathrm{F}\left(\mathrm{Y}_{\mathrm{B} 1}, \mathrm{Z}_{\mathrm{B} 1}\right)=\mathrm{a}_{5} \mathrm{X}_{\mathrm{B} 1}+\mathrm{b}_{5} \mathrm{Y}_{\mathrm{A} 1}+\mathrm{c}_{5} \mathrm{Z}_{\mathrm{A} 1}-\mathrm{d}_{5} \tag{10}
\end{equation*}
$$

The isoline of the surface defined by the function (10) is drawn for zero values of this function. The coordinates of the points on this isoline give a set of values of the variable parameters for which the system of equations has solutions. With these values and the functions (9), the corresponding set of calculation parameters is determined.

With $\mathbf{n}=\mathbf{6}$, there are five equations of type (5). In this case, the calculation parameters are determined from the first three equations as functions of the variable parameters, which are written in the form (9). With the remaining two equations, the two functions of the variable parameters are written:

$$
\begin{align*}
& \mathrm{F}\left(\mathrm{Y}_{\mathrm{B} 1}, \mathrm{Z}_{\mathrm{B} 1}\right)=\mathrm{a}_{5} \mathrm{X}_{\mathrm{B} 1}+\mathrm{b}_{5} \mathrm{Y}_{\mathrm{A} 1}+\mathrm{c}_{5} \mathrm{Z}_{\mathrm{A} 1}-\mathrm{d}_{5},  \tag{11}\\
& \mathrm{Q}\left(\mathrm{Y}_{\mathrm{B} 1}, \mathrm{Z}_{\mathrm{B} 1}\right)=\mathrm{a}_{6} \mathrm{X}_{\mathrm{B} 1}+\mathrm{b}_{6} \mathrm{Y}_{\mathrm{A} 1}+\mathrm{c}_{6} \mathrm{Z}_{\mathrm{A} 1}-\mathrm{d}_{6}
\end{align*}
$$

The isolines of the surfaces determined by the functions (11) are drawn for zero values of these functions. The coordinates of the intersection points of the two isolines determine the values of the variable parameters for which the system of equations (5) has a solution. With these values and the functions (9), the values of the calculation parameters are determined.
II.2. Synthesis algorithm designed for Watt-2023 programming environment.

1. Enter the relative linear displacements $\mathrm{S}_{1 \mathrm{i}}$ of the centre of the cylindrical pair $B$, the relative angular orientations $\theta_{1 i}$ of the coil 1 , the limits and steps of changing the variable parameters.
2. Enter the expressions (2) to determine the coordinates, and of the centre of the spherical pair B.
3. Enter the expressions (6) to determine the coefficients of the system of equations (5).
4. With $\mathbf{n}=\mathbf{5}$ and one variable parameter, the functions (7) and (8) are entered. The graphs of these functions are
drawn and the values of the variable and the calculation parameters are determined from them.
5. With two variable parameters, the functions (9) and (10) are introduced. The isoline of the surface defined by the function (10) is drawn for zero values of this function. With the coordinates of points of this isoline (which are values of the variable parameters) and the functions (9), the calculation parameters are determined.
6. With two variable parameters, the functions (9) and (11) are introduced. The isolines of the surfaces determined by the functions (11) are drawn. The coordinates of their intersection points (which give the values of the variable parameters) are calculated from them, and with them and the functions (9) the calculation parameters are determined.

## III. RESULTS AND DISCUSSION

Example: To synthesize an eight-link lever mechanism for moving loads between two belt conveyors at given: $y_{16}=0.6 \mathrm{~m}$; relative linear displacements $S_{12}=0.1 \mathrm{~m}, \quad S_{13}=0.18 \mathrm{~m}, \quad S_{14}=0.24 \mathrm{~m}, \quad S_{15}=0 \mathrm{~m} ; \quad$ relative angular orientations of the leading link $\theta_{12}=\pi / 3, \theta_{13}=2 \pi / 3$, $\theta_{14}=\pi, \theta_{15}=2 . \pi, \theta_{16}=\pi / 2$.

After carrying out the synthesis, plot the resulting eight link mechanisms, for the six pre-fixed positions where the errors are equal to zero (fig. 2 to fig. 7) of the mechanism to show the real results of the so-called marginal synthesis.

In positions $n=1$ to $n=6$, the error in the specific momentary centres of the synthesis, the error shown in figures fig.2, fig.3, fig.4, fig.5, fig. 6 and fig. 7 have zero values, and in the intervals between the points, the corresponding error is shown in the lower diagram on each figure in red color. The maximum synthesis error is between $\mathrm{n}=5$ and $\mathrm{n}=6$ and is 5.2 mm in size.

As a result of the computer analysis, 3700 variants were generated, and the optimal synthesized eight-link mechanism is number 1980. The program has calculated and can be seen from the bottom row all the


Fig. 2 First extreme remote position


Fig. 3 Second extreme remote position


Fig. 4 Third extreme remote position


Fig. 5 Fourth extreme remote position


Fig. 6 Fifth extreme remote position


Fig. 7 Sixth extreme remote position
kinematic parameters of the synthesized mechanism (movement of all links as coordinates, linear velocities and accelerations fig.8).


Fig. 8 Modification of the movement error of the mechanism

## IV. CONCLUSION

This article is intended for teaching bachelor's and master's students in the direction of mechanical engineering and the field of application synthesis of mechanisms and machines for industry. An illustrative drawing of such a mechanism is shown here in Fig.9.

The Watt porogram was used, which can be used to synthesize various types of mechanisms, both in infinitely close and infinitely distant positions of the executive unit. The classic Burmester principle was used as the basis of the synthesis, but adapted by the author for multi-link machines in industry.


Fig. 9 Illustrative scheme of a mechanism
On the condition that an eight-link guide mechanism is to be synthesized, it is most appropriate to synthesize a family of eight-link lever mechanisms, in which, after a subsequent kinematic analysis, the relative error in the
predetermined trajectory between points $1,2,3,4,5$ and 6 (fig.1). The author proposes a unified approach for the synthesis of such mechanisms on extremely distant discrete positions.

The example made shows the synthesis of an industrial sanitary ware load transfer manipulator, where the main problems are high velocity values at points 1 and 6 of the executive unit reactor. The linear accelerations at points 1, 2, 3, 4, 5 and 6 shown in Fig. 2, Fig. 3, Fig. 4 and Fig. 5 are also non-constant.

In order to carry out an optimal synthesis of the shown mechanism with subsequent kinematic analysis and establishing the effect of the synthesis, the following activities were carried out:

1. Fig. 1 shows an eight-link lever mechanism connected to an industrial manipulator, characterized by the fact that it intercepts a load in one plane and moves it to another parallel plane with a different deviation. As the position function of the original and reduced mechanism is identical.
2. Initially, the characteristic points $1,2,3,4,5$ and 6 are set. The Watt program is introduced and 3700 variants of eight-link mechanisms are synthesized. From them, after a thorough kinematic analysis, we choose the 1980 version.
3. Draw the kinematic diagrams for the same positions (Fig. 3, Fig. 4, Fig. 5, Fig. 6, Fig. 7 and Fig. 8).
4. The linear and angular velocities and accelerations of the characteristic points are compared, and in each figure the function of the change of the relative error of the executive unit between the output and the reduced mechanism is depicted.
5. The peak values of the linear and angular accelerations of the manipulator at characteristic points $1,2,3,4,5$ and 6 of the mechanism are minimal.
6. The author proposes a unified approach for the synthesis of eight-link mechanisms using the Watt synthesis program at finitely spaced discrete positions, which can be applied to various types of transport robots and manipulators, both for guides and for actuators.
7. The obtained numerical results were obtained theoretically and based on them a prototype was developed, which shows increased reliability during operation in real conditions, compared to that of existing models. The prototype of the mechanism is made in a real industrial environment.

The purpose of this publication is to raise the level of teaching in the field of the theory of machine mechanisms and automatic lines in industrial production.

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