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Property Insurance Decision-making on the Basis of Utility Functions

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***Abstract*. Property and other valuables insurance is widespread all over the world. An insurance company assumes the risk of damage or total destruction of the insured property. When this kind of damage or destruction is established, the company pays its client compensation (insurance premium) up to the amount specified in the insurance contract. For his part, the insured must pay a certain amount to the firm for the provision of insurance services. In any property insurance process, the question arises as to whether it is appropriate to insure the property for the price offered by the firm. The paper considers an approach to solving this problem based on expected utility theory.**

Keywords: Expected utility, insurance policy, insurance risks, risk attitude.

I. Introduction

The insurance industry is a powerful sector of financial operations in the world. Insurance companies offer insurance for property and health, industrial equipment of all kinds, vehicles and freight, passenger travel and flight safety, space launches, manned space flights, financial transactions, and much more.

There are two types of risk that must be considered in the insurance process. The first type describes the possible loss of the insured due to possible damage or total loss of their property. For the transfer of this type of risk to the insurance company, the insured pays a fee to the insurer. This payment is expressed in the form of the price of the insurance policy.

On the other hand, an insurance firm faces the risk of paying out large sums of money if its clients apply for insurance claims all at once. Insurance firms use proven methodologies to assess their risks and to price insurance policies for different insurance situations [1 - 6].

When deciding whether to insure their property, the insured must determine whether they are satisfied with the property insurance offered by the firm and the price of that insurance.

The simplest way to decide whether to insure a property is to compare the expected cash benefits with the price of the policy. However, in addition to the monetary value, the property may have an additional utility for the individual, which cannot always be expressed in terms of a simple monetary equivalent. Therefore, it is considered appropriate to value the insured property in terms of its usefulness to the individual.

The paper aims to demonstrate that expected utility theory can be used to make a decision about the purchase of property insurance policy. The task of the paper is to illustrate how a property insurance decision is made on the basis of the individual policy holder's utility function.

The method used in this paper is based on the use of reference lotteries to estimate the risk attitude of the insured. The analysis of insurance alternatives is the basis for making insurance decisions.

The paper examines an approach to property insurance based on the subjective utility function in expected utility theory.

II. A Brief Introduction to Expected Utility Theory

In 1738 D. Bernoulli put forward the ingenious idea that the utility of money does not increase in direct proportion to its quantity, but in a more complex way, namely as the logarithm of the quantity of money. Modern evidence shows that Bernoulli's conjecture of a logarithmic

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relationship between the utility of money and the quantity of money only occurs in certain specific situations.

Another important achievement of D. Bernoulli was the explicit notion that the utility of money depends not only on the additional amount of money, but also on the initial amount available to the individual.

Unfortunately, D. Bernoulli's fruitful ideas about utility have been forgotten for almost 200 years. A step in the direction of developing a grounded utility theory was the work of E. P. Ramsey [7] and B. de Finetti [8]. The first work that successfully laid the theoretical foundation for expected utility was the work of von Neumann J. and O. Morgenstern [9]. In this work, the authors proposed a system of axioms about an individual's preferences on a set of risky lotteries (games). They proved that by satisfying the requirements of these axioms, an individual's utility function can be constructed, and the best actions of an individual in risky choice situations are those that maximise the expected utility.

An essential feature of the von Neumann and Morgenstern approach was that the probabilities of relevant outcomes of lotteries (games) were assumed to be known and determined in an objective way. It is this approach that we use in the present paper.

In 1954, L. J. Savage [10] proposed another axiomatic basis for expected utility. In essence, Savage's approach is a synthesis of the ideas of de Finetti and von Neumann and Morgenstern. Its results consist in both a complete theory of subjective probabilities and a complete theory of expected utility.

Various aspects of the application of expected utility theory to insurance processes are discussed in [11 - 15]. Consider one important concept related to lotteries. The minimum fixed amount that is as attractive to an individual as participating in some lottery is called the deterministic equivalent of that lottery. The construction of an individual's utility function is based on his estimates of the deterministic equivalents of a sequence of lotteries given in the range of variation of some factor on which the utility function is defined. Money is most often used as such a factor.

The difference between the expected lottery winnings and the deterministic equivalent of that lottery for a particular individual is called the risk penalty, or risk premium. For risk-averse individuals, their risk penalty will always be positive. This paper assumes that all individuals making property insurance decisions are risk averse.

III. Analysis of the Properties of the Utility Function

Fig. 1 shows a graph of the conditional utility function, reflecting individual's perceptions of the utility of amounts of money in the range [0, 100] of conditional monetary units (c.u.). This graph reflects the subjective views of the risk-averse individual.

A basic property of any marginal utility function is that it reflects the relative reduction in utility as the value of the valuation criterion X increases. This property is visually illustrated in Fig. 1.

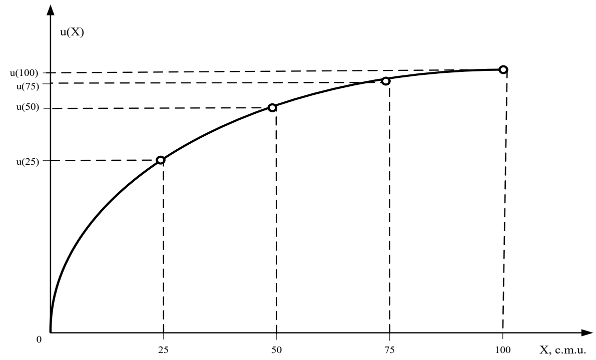


Fig. 1. Graph of the conditional utility function of the risk-averse individual.

On what factors does an individual's risk aversion in risky choice situations depend? Let us consider this problem in the context of expected utility theory using the example of choice in risky lotteries (games).

The main factors influencing the results of risky elections are:

- amplitude of variation of outcome estimates;
- probabilities of outcome estimates;
- the value of the rates when choosing in relation to an individual's available resources.

Let us analyse these factors in detail. Let us call an exact lottery (game) a lottery (game) that has two outcomes, the probability of each outcome being 0.5, and whose outcome estimates are equal in absolute value, but have opposite signs.

Suppose that the individual (the decision maker) has constructed his utility function on the set of some hypothetical sums of money X, whose graph is shown in Fig. 2. Suppose that an individual has a sum of money x_0 that constitutes his initial wealth. This individual is asked to choose between the following alternative actions:

1. To participate in the lottery L_1 : $(a, 0.5; -a, 0.5)$;
2. To participate in the lottery L_2 : $(2a, 0.5; -2a, 0.5)$;
3. To refuse to participate in both lotteries.

Suppose an individual decides to participate in the lottery L_1 . Then on condition of winning, his current wealth will be equal to $x_0 + a$; on condition of losing, his current wealth will be equal to $x_0 - a$. If the individual decides to participate in the lottery L_2, his current wealth will be equal to either $x_0 + 2a$ or $x_0 - 2a$. The outcomes of both lotteries are shown on the horizontal axis X in Fig. 2. The vertical axis shows the values of the utility function corresponding to the reference value x_0 and the outcomes of both lotteries. Let us calculate the expected values of lotteries L_1 and L_2 :

$$E(L_1) = 0.5(x_0 + a) + 0.5(x_0 - a) = x_0;$$
$$E(L_2) = 0.5(x_0 + 2a) + 0.5(x_0 - 2a) = x_0.$$

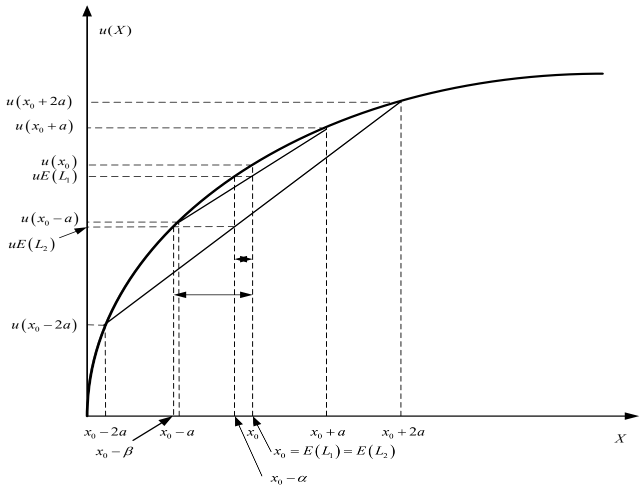


Fig. 2. An individual's utility function constructed for his cash and a reflection of two exact lotteries.

It is clear that in this situation the expected values of the lotteries cannot serve as a basis for choosing in favour of one of them.

The values of the expected utility for each of the lotteries can be calculated using the expressions (see Fig. 2)

$$Eu(L_1) = 0.5u(x_0 + a) + 0.5u(x_0 - a);$$
$$Eu(L_2) = 0.5u(x_0 + 2a) + 0.5u(x_0 - 2a).$$

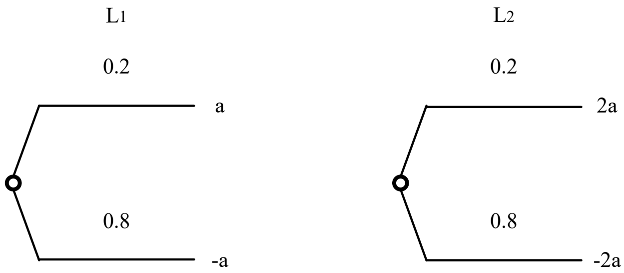
These expected utility values are shown on the vertical axis in Fig. 2.

Obviously, with this form of utility function (the individual is risk averse) $Eu(L_1) > Eu(L_2)$. This is explained as follows. The utility gain in the lottery L_1 on the interval $(x_0 + a)$ will be smaller than the utility loss on the interval $(x_0 - a)$ due to the property of relative utility loss with increasing values of X. This effect is even more pronounced in the lottery L_2 because of the larger scatter of the win/loss values relative to the reference value x_0.

In either case, the expected utility values of lotteries will be negative. Since the negative value $Eu(L_1)$ is less than the negative value $Eu(L_2)$, the individual should prefer participation in the lottery L_1 to the lottery L_2.

Not participating in lotteries leaves the individual with his or her own interests, because the expected utility of that action is 0. In general, risk increases when the amplitude of the variation in the estimates of outcomes in a risky lottery (game) increases.

Let us consider how the outcomes of risky choices can be affected by the probabilities of the outcomes. Let us turn to the lotteries discussed above. Suppose now that the lotteries are formulated as follows:



How would changing the probabilities of the outcomes affect the outcome of a choice between lotteries? Let us calculate the expected values of the lotteries.

$$E(L_1) = 0.2(x_0 + a) + 0.8(x_0 - a) = x_0 - 0.6a;$$
$$E(L_2) = 0.2(x_0 + 2a) + 0.8(x_0 - 2a) = x_0 - 1.2a.$$

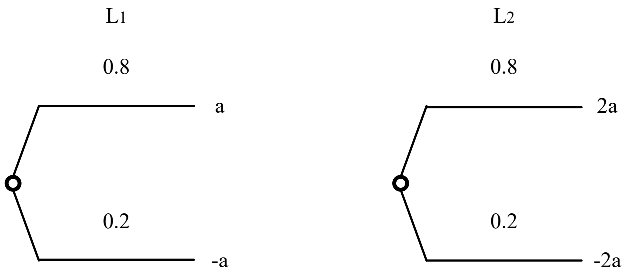
Even using the principle of maximisation of expected value and not involving the principle of maximisation of expected utility, we can confidently conclude that the lottery L_1 is preferable to the lottery L_2.

The expected utility values for these lotteries are calculated using the expressions

$$Eu(L_1) = 0.2u(x_0 + a) + 0.8u(x_0 - a)$$
$$Eu(L_2) = 0.2u(x_0 + 2a) + 0.8u(x_0 - 2a)$$

It is not difficult to show that the expected utilities of these lotteries are smaller than the expected utilities of the previously considered lotteries. This is because in the choice situation considered now, the utility values of the winnings are significantly reduced, but the utility values of the losses are significantly increased. As in the previous case, the lottery L_1 would be preferred to the lottery L_2. Now the utility of not participating in the lotteries has increased even more.

Let us consider another lottery option.



Let us calculate the expected values of the lotteries.

$$E(L_1) = 0.8(x_0 + a) + 0.2(x_0 - a) = x_0 + 0.6a;$$
$$E(L_2) = 0.8(x_0 + 2a) + 0.2(x_0 - 2a) = x_0 + 1.2a.$$

Using the criterion of maximising the expected value, it can be seen that the lottery L_2 is now preferred, and both lotteries are preferred over refusing to participate in them.

The expected utility values of these lotteries are calculated as follows:

$$Eu(L_1) = 0.8u(x_0 + a) + 0.2u(x_0 - a);$$

$$Eu(L_2) = 0.8u(x_0 + 2a) + 0.2u(x_0 - 2a).$$

Obviously, in this case $Eu(L_2) > Eu(L_1)$, from which it follows that participation in the lottery L_2 is preferable. It is also clear that under the criterion of maximising expected utility the least preferred option is not to participate in the lottery.

What general conclusion can be drawn from looking at these simple examples? The outcome of choices in risky situations will always be influenced by the probabilities of winning and losing. The greater the probabilities of winning (and therefore the lower the probabilities of losing), the more preferable the option becomes for the individual.

Consider the effect of initial wealth on an individual's (decision maker's) risk appetite. Fig. 2 shows two exact lotteries (games) for the initial wealth of the individual, valued by the sum of money x_0. Now suppose that the initial wealth is valued at $x_0' < x_0$. How will the individual's attitude to risk change in the situation of choosing on exact lotteries L_1', L_2', and the option of not participating in risky lotteries (games). If we plot the new choice situation in Fig. 2, it is obvious that the point x_0' will be to the left of the point x_0 on the horizontal axis. It is also obvious that the expected values of both lotteries will be equal to x_0' , so that the criterion of maximising the expected value is useless in this new choice situation.

Given the property of the relative diminishing marginal utility, it can be argued that the utility reductions of lotteries L_1', L_2', will be even larger, since the point x_0' corresponds to a region of more rapid utility changes. Accordingly, the positive utility of the lotteries will also increase. But the overall effect of these changes would be to reduce the expected utility for both lotteries, with the greater utility reduction corresponding to the lottery L_2'. Therefore, as before, the lottery L_1' would be preferable to the lottery L_2', and not participating in both lotteries would be the most preferable course of action for the individual.

Now let the initial wealth of an individual, all other things being equal, be the sum of $x_0'' > x_0$ .This point will lie to the right of the point x_0 in Fig. 2. Obviously, now the changes in the utilities of the wins and losses in the new lotteries will be smaller than the changes in the corresponding utilities in the previous lotteries. This leads to a reduction in the expected utility of both lotteries. Although in this case the individual should prefer not to participate in both lotteries, however, the degree of this preference becomes smaller. It is quite possible that, if the value is large enough, the individual will agree to participate in one of the lotteries. This indicates his greater risk appetite in this choice situation.

How can the degree of risk aversion of a decision maker be formally assessed if his utility function on the set of values of the evaluation criterion X is constructed? To simplify things further, assume that X represents some amount of money that an individual has or could potentially have. This approach of reasoning on sums of money is prevalent in works on expected utility. The reason for this is the clear interpretability of the results achieved on hypothetical lotteries (games). The conclusions drawn from further analysis are fully transferable to any measurement scale of evaluation criterion X .

The most appropriate measure for formally assessing risk attitude is *the Arrow-Pratt absolute risk aversion measure*

$$r(X) = -\frac{u''(X)}{u'(X)}. \qquad (1, a)$$

This expression is general in nature. If the evaluation is performed for a particular value $x_0 \in X$, expression (1, a) takes the form

$$r(x_0) = -\frac{u''(x_0)}{u'(x_0)}. \qquad (1, b)$$

The second derivative $u''(x_0)$ characterises the degree of curvature of the utility function $u(X)$ at the point $x = x_0$. For the risk-averse decision maker, this derivative will always be negative at all points of definition of the function $u(X)$. The first derivative $u'(x_0)$ shows the slope of the function's $u(X)$ graph relative to the horizontal axis X at the point $x = x_0$. For a risk-averse decision maker, $u'(X)$ will be positive at all defining points of the function $u(X)$. To ensure that the values of $r(X)$ are always positive in expressions (1 a, b), the "-" sign is introduced. Higher values $r(X)$ correspond to lower risk aversion, and lower values $r(X)$ correspond to higher risk aversion.

IV. Making Decision to Insure Property

Let us consider solving this utility function problem using the following illustrative example. An individual's current wealth is valued at CU 100,000. This includes movable and immovable property and cash. The individual's immovable property is valued at 30,000 c.m.u. and the probability of losing it completely within the next year is estimated at 0.1. The individual wishes to insure the immovable property for an amount not exceeding 3,000 c.m.u. It must be verified whether this is the optimal solution, given that his utility function on the current wealth is shown by the logarithmic function $u(X) = \ln(X)$.

Let us construct a graph of the individual's utility function. We do not need to construct this graph for the whole set X, it is enough to construct it only in the vicinity of the boundary amount of 100,000 c.m.u. A fragment of the graph is shown in Fig. 3.

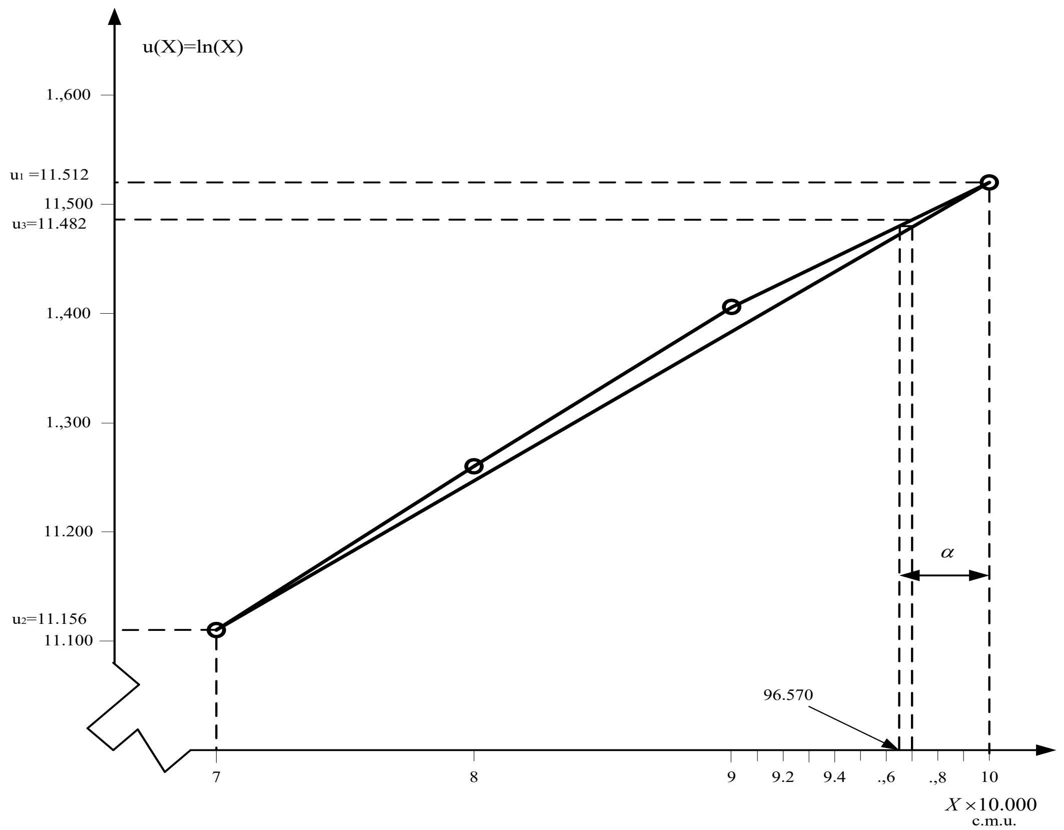


Fig. 3. Fragment of the utility function $u(X) = \ln X$ of an individual deciding to insure property.

$$\ln(100,000 - \alpha) = 11.478$$

Let us solve this equation relative to α

$$100,000 - \alpha = e^{11.478} = 96,567 .$$

$$\alpha = 100,000 - 96,567 = 3,432 \text{ c.m.u.}$$

From this it follows that if an insurance company offers to sell a property insurance policy for a price lower than 3,432 c.m.u., it is profitable for the individual to purchase the policy.

In this example, to ensure clarity and simplify calculations, we have used the utility function in logarithmic form $u(X) = \ln X$. In the case of an arbitrary form of the utility function graph, which is the case in most practical applications, the required utility values can simply be read from the graph.

V. Conclusion

To avoid the risks associated with damage or total destruction of property belonging to them, people take out insurance. The insurance company assumes the risk associated with the condition of the property during the insurance period and pays compensation in cases specified in the insurance contract.

When deciding on property insurance, the insurer must consider many factors including the importance of the insured property to the insured person, uncertainty about the possible future state of the property, as well as insurance costs (insurance contract price).

The use of a utility function constructed on the interval of the monetary equivalent of the relevant property allows us to explicitly represent the individual's subjective judgments about the utility of that property. If an individual has subjective judgements about the chances of damaging or losing a property in the future, the task of deciding whether to insure that property becomes trivial. Using the methodology presented in the previous section, an individual can estimate his financial possibilities associated with insurance, compare these possibilities with the price of the insurance policy, and on this basis make a final decision regarding insurance.

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