

same velocity as the velocity of a point from the cutting edge of the cutting disc. This is perfectly acceptable, subject to the condition that the contact size is always smaller than the radius R of the cutting disc.

We determine the time for cutting up the optical slug both under traditional conditions of machining (τ_{cm}) and when applying vibrational cutting (τ_{ep}), and quantitatively evaluate the degree of influence of the forced oscillations on the increase in cutting intensity, which we estimate according to the expression

$$\eta = \left(1 - \frac{\tau_{ep}}{\tau_{cm}}\right) 100, \quad (1)$$

assuming that $\tau_{cm} > \tau_{ep}$.

We connect the sample with a coordinate system x, y , as shown in Fig. 1. Based on the theory of mechanical failure of brittle materials, we express the amount of the work, expended for cutting under traditional machining conditions for time Δt by the expression

$$\Delta W_{up} = \alpha j_{cm} V_0 \Delta t, \quad (2)$$

where V_0 is the velocity of the disc periphery (the cutting speed); j_{cm} - an impulse, transmitted to the optical sample parallel to the x axis per unit of time (the average interaction force);

α - a coefficient, characterizing part of the work, done by the forces during cutting, spent on destroying the optical workpiece ($\alpha < 1$), (selected from a table).

The direction of the forces, performing work in traditional cutting of optical slugs should coincide with the forces, causing destruction of the workpiece surface.

The magnitude of ΔW_{up} should match the work to destroy the sample and is equal to

$$\Delta W_{paz} = \gamma_{cm} h l(t) \dot{x}(t) \Delta t, \quad (3)$$

where γ_{cm} is the specific work for destruction of the material when cutting; h - width of cut; $l(t)$ - size of the cut low along the axis, equal to $l(t) = 2x(t)$ for the chosen shape of the slug, at $0 < x(t) < b/2$, where b is the depth of the cut.

For the purpose of finding the analytical solution of (2), we assume that j_{cm} is a constant, independent of the depth of the cut of the slug.

Then, from (2) and (3) the differential equation is obtained:

$$x(t) \dot{x}(t) = \frac{\alpha j_{cm} V_0}{2 \gamma_{cm} h}. \quad (4)$$

Considering the magnitude of γ_{cm} as constant and integrating (4) with the initial conditions $x(0) = 0$, we obtain the law of motion of the cutting line relative to the optical slug

$$x(t) = \beta_{cm} \sqrt{t};$$

where

$$\beta_{cm} = \sqrt{\frac{\alpha j_{cm} V_0}{\gamma_{cm} h}}. \quad (5)$$

Dependence (5) is valid for the interval $0 \leq t \leq \tau$, where τ is the time to cut half of the optical slug. It is determined by the equation of the type

$$x(t) = \frac{b}{2} = \frac{a}{\sqrt{2}}; \quad \tau = \frac{a^2}{2\beta_{cm}^2} = \frac{2\pi N}{\omega}, \quad (6)$$

where N is the number of disc revolutions for the time of cutting;

ω - its angular velocity.

At $t > \tau$ in the expression (3) it should be assumed

$$l(t) = 2[b - x(t)], \quad (7)$$

from where we obtain the differential equation

$$[b - x(t)] \dot{x}(t) = \frac{\alpha j_{cm} V_0}{2 \gamma_{cm} h}, \quad \tau < t < \tau_{cm}. \quad (8)$$

For the initial condition $x(\tau) = b/2$ for the solution of (8) is written

$$x(t) = \frac{b}{2} \left[2 - \sqrt{1 - \frac{4\beta_{cm}^2}{b^2} (t - \tau)} \right], \quad \tau < t < \tau_{cm}. \quad (9)$$

The time for the complete cutting the optical slug up τ_{cm} is determined by the condition $x(t) = b$, then from (6) we find

$$\tau_{cm} = \tau + \frac{b^2}{4\beta_{cm}^2} = \tau + \frac{a^2}{2\beta_{cm}^2} = \frac{a^2}{\beta_{cm}^2} = 2\tau. \quad (10)$$

According to formulas (5) and (9), the dependence $x(t)$ is antisymmetric (odd) with respect to the points $t = \tau, x = b/2$.

To calculate the work, spent on cutting up half of the optical slug, the expression (3) should be summed

$$\begin{aligned} \Delta W_{\text{paz}} &= \int_0^{\tau} \gamma_{cm} h l(t) \dot{x}(t) dt = 2\gamma_{cm} h \int_0^{\tau} x(t) \dot{x}(t) dt = \\ &= \gamma_{cm} h x^2(\tau) = \frac{\gamma_{cm} h a^2}{2} \end{aligned} \quad (11)$$

When cutting under traditional conditions, when a static load P_{cm} is acting, the magnitude of the impulse per unit of time is equal to the period of the forced vibrational oscillations T and is determined

$$I_{cm} = \frac{j_{cm}}{T} = \frac{\omega_{ep}}{2\pi} \int_0^{\tau} F_{cm} dt = F_{cm}; \quad \beta_{ep} = \sqrt{\frac{\alpha F_{cm} V_0}{\gamma_{cm} h}}, \quad (12)$$

where ω_{ep} is the circular velocity of the vibrational oscillations.

Then, the time to cut up the optical slug under traditional processing conditions according to (10) will be equal to

$$\tau_{cm} = \frac{a^2 \gamma_{cm} h}{\alpha F_{cm} V_0}. \quad (13)$$

The time to cut up optical slugs under the condition of vibro-impact cutting is determined, assuming that the vibro-impact oscillations are transmitted to the slug in its lower part below the x axis (Fig. 1). The vibro-impact mode of interaction between the periphery of the cutting part of the disc and the machined surface of the slug is realized in addition. In this case, for time T , equal to the period of the forced oscillations, the mechanical interaction between the slug and the cutting disc constitutes only the part t_k of it, and in the remaining time $T - t_k$ we have an absence of cutting. Therefore, the chipping of diamond particles from the cylindrical surface of the disc results from the pulse impact with a frequency, equal to the frequency of rotation of the electric motor of the centrifugal vibrator, creating the forced oscillations.

Thus, in order to determine the time of cutting the optical workpiece under an introduced vibrational impact, the following correction should be made in the expressions, obtained during processing under traditional conditions: firstly, instead of I_{cm} , the magnitude of the average period of oscillation T should be taken, and the impulse of the forces of interaction between the peripheral surface of the cutting disk and the slug surface I_{ep} should be accepted; secondly, instead of the magnitude of the specific work for destruction under traditional conditions γ_{cm} , it should be assumed that its magnitude corresponds to the vibro-impact mode of cutting γ_{ep} .

Taking into account the above, expressions (12) and (13) in the conditions of a vibro-impact mode of cutting take the form

$$\Delta W_{\text{ycm.ep}} = \alpha V_0 j_{ep} \Delta t, \quad (14)$$

$$\Delta W_{\text{paz.ep}} = \gamma_{ep} h l(t) \dot{x}(t) \Delta t, \quad (15)$$

where $\Delta W_{\text{ycm.ep}}$ and $\Delta W_{\text{paz.ep}}$ are respectively the work, spent on cutting and the work on destructing the sample in the vibro-impact mode of processing.

Then the differential equation (14) and its solution (15) take the form:

$$x(t) \dot{x}(t) = \frac{\alpha j_{ep} V_0}{2\gamma_{ep} h} \quad (16)$$

$$x(t) = \beta_{ep} \sqrt{t}, \quad \text{където } \beta_{ep} = \sqrt{\frac{\alpha j_{ep} V_0}{\gamma_{ep} h}} \quad (17)$$

After the changes, made in expressions (8) and (9), the time for cutting the workpiece under vibro-impact action on it, will be equal to:

$$\tau_{ep} = \frac{a^2 \gamma_{ep} h}{\alpha I_{ep} V_0}. \quad (18)$$

Substituting dependencies (13) and (18) into (1), we will obtain an expression for determining the degree of increase in the intensity of cutting optical slugs

$$\eta_{\tau} = \left(1 - \frac{\tau_{ep}}{\tau_{cm}}\right) = \left(1 - \frac{F_{cm} \gamma_{ep}}{j_{ep} \gamma_{cm}}\right), \quad (19)$$

where j_{ep} is the impulse of the forces on the optical slug during the time of its contact with the cutting disc.

It follows from (19) that, other things being equal, the degree of the intensifying effect of vibration oscillations on the productivity of the process of mechanical cutting of optical materials is determined by two ratios: 1. the ratio of the specific work for destruction of the workpiece surface in the vibro-impact mode of cutting to the work, done under traditional conditions ($\gamma_{ep} / \gamma_{cm}$); and 2. the ratio of the force of the impulses for the period of oscillations under traditional cutting conditions ($F_{cm} \cdot T$) to the force impulse in the transmission of oscillations to the machined workpiece for the time of contact between the slug surface and the cylindrical surface of the disc j_{ep} .

If we assume that $\gamma_{ep} = \gamma_{cm}$, then the assessment of the degree of influence of the forced oscillations on increasing the intensity of cutting optical materials can be calculated by the expression:

$$\eta_{\tau} = \left(1 - \frac{F_{cm} T}{j_{ep}} \right). \quad (20)$$

From analytical studies [1], using original mathematical models, the expression for determining the magnitude of j_{ep} in the case of a vibro-impact mode of cutting was obtained, which has the form

$$j_{ep} = \frac{8F_{cm}^2 \pi}{A_0 \omega_{ep} c} C.D, \quad (21)$$

where F_{cm} is the static effort; A_0 - the amplitude of the vibrational oscillations; c - hardness/stiffness of the weightless vibro-element, accepted in the models; C and D – dimensionless coefficients, which can be assumed to be equal to unity, or to be more than a unity.

These coefficients will be equal to unity in continuous modes of operation of the vibrator, when $A_0 = 2x_{cm}$, where $x_{cm} = F_{cm} / c$ is the value of the preliminary tightness in the system, determined by the static load.

Substituting the expression (21) into (20) and transforming, we obtain:

$$\eta_{\tau} = \left(1 - \frac{A_0}{2x_{cm} C.D} \right). \quad (22)$$

III. CONCLUSION

Based on the developed vibro-impact mathematical models, describing the process of cutting optical materials, a quantitative assessment of the degree of influence of the vibrational oscillations on increasing the intensity of cutting such materials can be made. With the increase in the amplitude of the vibrational oscillations, increase in the productivity of the process of cutting optical slugs is registered.

The obtained theoretical assessment of the increase in the process productivity is valid provided that the cutting resistance forces of, acting in the process of machining, are not taken into account. Taking into account the shape of the cutting disc, we can conclude that as the depth of its cut into the optical slug increases, the wedging forces, trying under the action of internal stresses to push aside the cut-off part of the workpiece, will increase. The exact evaluation of the influence of the different vibro-impact cutting modes on the productivity of machining can be performed after determining the dependencies, describing the change in the magnitude of the pulses j_{cm} and j_{ep} , transmitted to the machined samples parallel to the axis x per unit of time, while taking into account the change in the wedging forces along the depth of the cut workpiece.

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