On the Issue of Planning Sowing Agricultural Crops with the Minimum Risk under the Presence of Various Agroclimatic Conditions

Sharif E. Guseynov
Faculty of Science and Engineering
Liepaja University
Liepaja, Latvia
sh.e.guseinov@inbox.lv

Sergey I. Matyukhin
Department of Technical Physics and
Mathematics
Orel State University
Orel, Russia
sim1@mail.ru

Misir J. Mardanov
Institute of Mathematics and
Mechanics
Azerbaijan National Academy of
Sciences
Baku, Azerbaijan
misirmardanov@yahoo.com

Jekaterina V. Aleksejeva
Institute of Science and Innovative
Technologies
Liepaja University
Liepaja, Latvia
Riga Secondary School 34
Riga, Latvia
jekaterina.v.aleksejeva@gmail.com

Olga V. Sidorenko
Department of Accounting and
Statistics
Orel State Agrarian University
Orel, Russia
sov1974@mail.ru

Abstract - **The present paper deals with one problem of quantitative controlling the seeding of the sown area by agricultural crops in different agroclimatic conditions. The considered problem is studied from the standpoint of three strategies: from the seeding planning perspective aiming at minimal risk associated with possible unfavourable agroclimatic conditions (a probabilistic approach is used); from the perspective of obtaining the maximum crops sales profit (a deterministic approach is used); from the perspective of obtaining the maximum crops harvest. For the considered problem, mathematical models are constructed (one probabilistic model and two deterministic models, respectively), their analytical solutions are found, and then, using a specific example, the application of the constructed and solved mathematical models is illustrated as well as the obtained numerical results are analysed..**

Keywords - Agricultural crops, agroclimatic conditions, maximum profit, minimal risk

I. INTRODUCTION

One of the main branches of agriculture is farming – the use of land for the purpose of growing crops which will be discussed in this paper. Depending on soil and climatic conditions, cropping/farming is divided into the following categories: land reclamation (melioration farming); irrigated cropping; dry farming. Most countries of the European Union, including Latvia, have irrigation farming [1]. The main indicators of soil fertility necessary for the formation of high yields of crops are agrophysical indicators (basic: density, porosity, fine-grained structure, water-strength structure), biological indicators (basic: the presence of organic matter, including humus, phytosanitary state, biological activity, enzymatic activity) and agrochemical indicators (absorption capacity, soil reaction (pH), the presence of nutrients). Crop yields are very sensitive not only to soil fertility indicators, but also to climatic indicators, the main of which are temperature-humidity and temperature-wind (taking into account radiation) indicators. Together these indicators are called soil-climatic indicators (conditions), and it is these conditions that determine the success or failure of all stages of the process of growing crops [2]-[12]. The yield of crops by country, depending on soil and climatic conditions, crop farming, as well as macroeconomic conditions, is quite different. Even in the countries of the European Union, there are noticeable differences both in the yield of crops and in the costs of growing them, and, consequently, in their costs to the final consumer [13]-[17]. In addition,

different crops respond differently to the same soil and climatic conditions that occur during a particular growing season. Timely and efficient sowing provides the one of principal earnests of a successful harvest of agricultural crops. In its turn, for timely and efficient sowing it is required up-to-date seasonal forecasting information which enables growers to plan their crop production from seeding to harvest. By now, the relevant government departments of the most agriculturally developed countries (which are characterized by so-called high-commodity agriculture) are diligently encouraging scientists to carry out basic research using methods of mathematical modelling into various issues and aspects of the agronomic requirements of crops to provide to crops' growers rigorously scientifically based guidelines on sowing and its timing, on machinery, on environmental conditions, on paddock preparation, etc. [18]-[24] (see also relatively old works [25]-[32] and relevant references therein).

In the present paper, we consider and investigate one agricultural problem staying within the frames of answering the only question: if the agrochemical, landscape-ecological, ecological-genetic, etc. characteristics of the sown field satisfy the necessary requirements for sowing and growing certain kinds of field crops, which crops and in what proportions should be sown in order to obtain the best harvest? This is done regardless of what uncontrollable and/or poorly controlled scenarios (for example, agroclimatic conditions; fight against plant diseases and pests, etc.) will take place at every stage of the sowing process of these crops (the sowing process is a whole set of necessary measures: [33]-[41]). In the present paper, by the term "best harvest" we will alternately (i.e. in the sense of logical disjunction "or") mean three quantitative values: (A) the least risks from a possible loss of harvested crop (for this purpose, the constructed mathematical model has a probabilistic character); (B) the maximum profit from the sale of the harvested crop (for this purpose, the constructed mathematical model has a deterministic nature); (C) guaranteed maximum harvest.

II. Statement of the Considered Problem, its Formalization, and Construction of a Mathematical Model from the Ensuring Minimal Risks Standpoint

Let us suppose that some agricultural enterprise has a sowing field, which according to its' agrochemical, landscape-ecological, ecological-genetic, etc. characteristics is suitable for growing several kinds of field crops, the yield of each depends on uncontrollable and/or poorly controlled factors, which as a whole we will simply call a set of scenarios (for example, climatic conditions; control of diseases and pests; etc.). Provided that the market demand for each of the harvested crop kinds is unlimited, it is required to determine which kind of crop and in what proportions should be sown in order to obtain a guaranteed yield that meets the goal of an agricultural enterprise: as it has been already mentioned in the Introduction section, the goal of an agricultural enterprise will be consistently taken as (A) the lowest risks from a

possible loss of crop yields, (B) the maximum crops sales profit, (C) the maximum crops harvest.

Now let us work out in detail and formalize the conceptually formulated problem. For this let us introduce the following designations: S ha is the area of the cultivated field; M is a number of crop kinds that the agricultural enterprise plans for seeding; c_i euro/ha is the total cost for all stages of the cultivation process for the i-th $\left(i = \overline{1,M}\right)$ kind of crop per 1 ha of the sown field; p_i euro/quintal is the forecasting market price for 1 quintal of the future harvest of the i-th crop kind; x_i ha is the required area of the sown field assigned to the i-th kind of crop. In addition, it is assumed that there is empirical data on the harvest (quintal/ha) for each crop kind for a certain period of time: $\left\{q_{i,j,k}\right\}_{j=\overline{1,N_i};k=\overline{1,K}}$ for $\forall i = \overline{1,M}$, where N_i means the number of years for which statistics on the yield by the kind of crop has been collected. Fields (including the current cultivated field under consideration) with more or less similar soil characteristics under different agroclimatic conditions: see [2], [3], [36], [40] are considered. Using statistical methods, having properly processed [42]-[47] the known empirical data $\left\{q_{i,j,k}\right\}_{j=\overline{1,N_i};k=\overline{1,K}}$, it is necessary to determine the following numerical characteristics for each kind of M kinds of crops: sample mean $\left\{m_i\right\}_{i=\overline{1,M}}$, biased sample variance $\left\{\sigma_i^2\right\}_{i=\overline{1,M}}$, unbiased sample variance $\left\{\dfrac{N_i}{N_i-1}\sigma_i^2\right\}_{i=\overline{1,M}}$. It is important to note that in order to obtain adequate values $\left\{m_i\right\}_{i=\overline{1,M}}$, $\left\{\sigma_i^2\right\}_{i=\overline{1,M}}$, the required proper statistical processing of the available empirical data $\left\{q_{i,j,k}\right\}_{j=\overline{1,N_i};k=\overline{1,K}}$ implies the statistical processing of information on micro- and macro-fluctuations (a) key soil quality indicators (agrophysical, biological and agrochemical indicators: see Introduction); (b) hydro-meteorological parameters (amount of precipitation, moisture reserves in the soil, water evaporation, air humidity, air temperature, soil temperature, wind speed, solar activity, aridity, floods); (c) the spread of diseases and pest proliferation; (d) organizational and technological conditions (seed quality, sowing time, crop rotation and predecessors choice, tillage, mineral fertilizers, crop protection agents, etc.).

Let us begin constructing a mathematical model of a formulated and formalized agrarian problem, in which goal (A) is chosen as the optimized goal – achieving the least risks from a possible loss of productivity of cultivated M kinds of crop species. Using introduced designations, it is possible to write that the total profit (euro) of an agricultural enterprise after the sale of the entire harvested crop is determined by the expression

$$R(x) \overset{def}{\equiv} \sum_{i=1}^{M} \left(p_i m_i - c_i\right) x_i. \tag{1}$$

From (1) it directly follows that if for any j-th $\left(j \in \{1,\ldots,M\}\right)$ kind of agricultural crop $p_j m_j - c_j \leq 0$, is valid, then seeding of this kind of crop species either is not reasonable (for $p_j m_j - c_j = 0$), or will result in revenue losses (for $p_j m_j - c_j < 0$). However, in the agricultural industry, as in many industries connected with production or mining, there could be faced scenarios where an enterprise must proceed with production, even acknowledging the fact that the sale of goods will not bring any profit or even will incur some losses. For example, if a procurement contract is concluded between an agricultural enterprise and a buyer (for example, the government purchasing agent), under the terms of which the agricultural enterprise is obliged to transfer a certain part of the harvested crop to the buyer-procurer at a different price $\hat{p}_i$ euro/quintal (the so-called a purchasing price), which does not exceed the market price: $\hat{p}_i \leq p_i, i \in \{1,\ldots,M\}$. In this case, instead of formula (1), we have the formula

$$R(x) = \sum_{i=1}^{M} \left(p_i m_i - c_i\right) x_i + \sum_{i=1}^{M} \left(\hat{p}_i - p_i\right) Q_i, \quad (2)$$

where $Q_i \geq 0$ denotes the volume (quintal) of the harvested crop for the i-th kind of crop species, which is agreed in the procurement contract between the buyer and agricultural enterprise. Obviously, if for any j-th $\left(j \in \{1,\ldots,M\}\right)$ kind of agricultural crop the inequality $p_j m_j - c_j \leq 0$ takes place, then for an agricultural enterprise the following plan will be the best: $x_j = \dfrac{Q_j}{m_j}$.

Remark 1. If there is no obligation of selling a part of the harvest of any k-th $\left(k \in \{1,\ldots,M\}\right)$ crop species in the procurement contract, then it is obvious that $Q_k = 0$ in formula (2); if there is no such procurement contract concluded, then the second term of formula (2) is identically equal to zero and, therefore, formulas (1) and (2) will coincide. Obviously, the conditions $p_k m_k - c_k > 0$ and $Q_k > 0$ generate an inequality constraint $m_k x_k \geq Q_k$ for the k-th kind of crop species, and the conditions $p_k m_k - c_k \leq 0$ and $Q_k > 0$ generate an equality constraint $m_k x_k = Q_k$. End of Remark (EOR)

Remark 2. In the 50-70s years of the XX century, when the rapid development of computer systems contributed to the unprecedented application of mathematical modelling to real problems of technical, environmental, economic and other systems [48]-[55] having different levels of complexity, there have been published multiple works on various problems of agriculture, in particular, on agro-economic problems (see [28], [30]-[32], [35], [36], [38] and relevant references therein), in almost all of which the harvest distribution density of most agricultural crop

species has been a priori considered having a Gaussian distribution. However, according to the numerous results obtained with the help of the "ZONT" aerospace sensing and technology (about 90% of the large-scale predictions made in advance by "ZONT" were later confirmed), the development of which began in the 80s of the 20th century in the Laboratory of Long-Term Forecasts of the Voronezh Agrarian University under the leadership of an authority scientist – Professor Isaak Beniaminovich Zagaytov, now we can state with a high degree of confidence that the series of agricultural crops yields are not subject to the Gaussian probability distribution [56]-[60]. Major portion of the series of agricultural crops yields can be described using semi-stable distribution laws [61], in particular, the Lévy-Zipf's law, which arises, as a rule, when studying complex systems with feedback. Further, considering the seasonally theoretical/true yields of an i-th $\left(i = \overline{1,M}\right)$ kind of agricultural crop as the values of some random variable with a given distribution density $f_i\left(\text{yield}_i\right)$, for example, of a Gaussian function $f_i\left(\text{yield}_i\right) = \dfrac{1}{\sqrt{2\pi\sigma_i^2}} e^{-\frac{\left(\text{yield}_i - m_i\right)^2}{2\sigma_i^2}}$,

we can interpret the parameters m_i and σ_i, which, as mentioned above, are found by proper statistical processing of empirical data $\left\{q_{i,j,k}\right\}_{j=\overline{1,N_i}; k=\overline{1,K}}$, as a sample mean and biased sample variance of this random variable, respectively. Then, in view of the fact that the total profit of agricultural enterprise

$$R\left(\text{yield}; x\right) = \sum_{i=1}^{M} \left(p_i \text{yield}_i - c_i\right) x_i + \sum_{i=1}^{M} \left(\hat{p}_i - p_i\right) Q_i, \quad (3)$$

which itself is a random variable, appears to be a linear function with respect to theoretical yields $\text{yield} = \left\{\text{yield}_i\right\}_{i=\overline{1,M}}$, then it is possible to find the distribution density of the random variable (3) in explicit form [62]. In particular, if we assume that the yields of all M kinds of crops do not depend on each other (in fact, this is not always the case!), then $f_R\left(\text{yield}\right) = \prod_{i=1}^{M} f_i\left(\text{yield}_i\right)$. It is important to note here that if, on a particular sown field, the cultivation of one of the M kinds of crops affects the cultivation of at least one of the other M kinds of crops, then even if we assume that the yield of each of these two kinds of crops has a normal distribution, the statement about that the linear combination of the yields of these two kinds of crops also has a normal distribution [63] (the well-known Cramér's decomposition theorem for a normal distribution, predicted in [64] and proved in [65], requires that the components involved in this decomposition are independent random variables). In other words, it is impossible to assert that if the yield of each kind of M agricultural crops is distributed normally, then the profit distribution density f_R (3) of an agricultural enterprise has the form of a Gaussian function with a location parameter

$\sum_{i=1}^{M}(p_i x_i) m_i - \sum_{i=1}^{M} c_i x_i$ and a scale parameter $\sum_{i=1}^{M}(p_i x_i)^2 \sigma_i^2$.

Taking into account the abovementioned circumstance is all the more important when the yields of agricultural crops M are distributed by other laws (the same or, even more complicated, different laws) EOR

Let us get back to formula (2) and, nevertheless, we will proceed from the fact that all M kinds of agricultural crops considered in this work are such that the value

$$\sigma^2 \overset{def}{\equiv} \sum_{i=1}^{M}(p_i \sigma_i x_i)^2 \qquad (4)$$

is the variance of a random variable (3) – the total profit of an agricultural enterprise from the sale of the entire harvest of agricultural crops, taking into account the existing procurement contract (see Remark 1). Since the value (4) can be interpreted as the risks of an agricultural enterprise from the loss of yields, the problem of minimizing these risks is reduced to the problem of minimizing the value (4), but now as a function of variables $\{x_i\}_{i=\overline{1,M}}$, where x_i means the required area (ha) of the cultivated field, which must be allocated under the i-th kind of crops species. In other words, we have the following one-criterion constrained optimization problem:

$$\underset{x \in \Omega}{\text{minimize}} \left[\sigma^2(x) \overset{def}{\equiv} \sum_{i=1}^{M}(p_i \sigma_i x_i)^2 \right], \qquad (5)$$

where

$$\Omega \overset{def}{\equiv} \left\{ x \in \mathbb{R}_+^M : \sum_{i=1}^{M} x_i = S, \ m_i x_i \geq Q_i, \ \forall i = \overline{1,M} \right\},$$

$$\mathbb{R}_+^M \overset{def}{\equiv} \left\{ x \in \mathbb{R}^M : x_i \geq 0, \ \forall i = \overline{1,M} \right\}.$$

Let us apply the Lagrange multipliers method to problem (5). Let us compose the Lagrange functions and find its stationary point using the Karush-Kuhn-Tucker conditions:

– Lagrange function:

$$L(x;\lambda) \overset{def}{\equiv} \sum_{i=1}^{M}(p_i \sigma_i x_i)^2 + \alpha \left(\sum_{i=1}^{M} x_i - S \right)$$
$$- \sum_{i=1}^{M} \beta_i (m_i x_i - Q_i); \ \alpha \in \mathbb{R}^1; \ \beta_i \in \mathbb{R}_+^1, \ \forall i = \overline{1,M}; \qquad (6)$$

– Karush-Kuhn-Tucker conditions for an extremum:

$$\begin{cases} \dfrac{L(x;\lambda)}{\partial x_i} = 0, \forall i = \overline{1,M}, \\[2mm] \dfrac{L(x;\lambda)}{\partial \alpha} = 0, \\[2mm] \beta_i \dfrac{L(x;\lambda)}{\partial \beta_i} = 0, \ \beta_i \geq 0, \forall i = \overline{1,M} \end{cases} \Leftrightarrow$$

$$\begin{cases} 2 p_i^2 \sigma_i^2 x_i + \alpha - \beta_i m_i = 0, \forall i = \overline{1,M}, \\[2mm] \sum_{i=1}^{M} x_i - S = 0, \\[2mm] \beta_i (Q_i - m_i x_i) = 0, \beta_i \geq 0, \forall i = \overline{1,M}. \end{cases} \qquad (7)$$

Having solved this system, we find the coordinates of the sought-for stationary point:

$$x_i^* = \frac{S}{p_i^2 \sigma_i^2 \sum_{j=1}^{M} \dfrac{1}{p_j^2 \sigma_j^2}} > 0, \ \forall i = \overline{1,M}.$$

By adding to this formula the condition $x_i = \dfrac{Q_i}{m_i}$ if $p_j m_j - c_j \leq 0$, we obtain the final formula for the coordinates of the sought-for stationary point $x^* \in \mathbb{R}_+^M$:

$$x_i^* = \begin{cases} \dfrac{S}{p_i^2 \sigma_i^2 \sum_{j=1}^{M} \dfrac{1}{p_j^2 \sigma_j^2}} & if \quad p_i m_i - c_i > 0, \\[6mm] \dfrac{Q_i}{m_i} & otherwise \end{cases} \qquad (8)$$

for $\forall i = \overline{1,M}$.

Further, since $\dfrac{L^2(x;\lambda)}{\partial x_i \partial x_j} = \dfrac{L^2(x;\lambda)}{\partial x_j \partial x_i} \ \forall i,j = \overline{1,M}$ and $\dfrac{L^2(x;\lambda)}{\partial x_i \partial x_j}\bigg|_{x=x^*} = \begin{cases} 2 p_i^2 \sigma_i^2 > 0 & if \quad i = j, \\ 0 & otherwise, \end{cases}$ then all of the leading principal minors of the Hessian matrix $H(L) = \left[\dfrac{L^2(x;\lambda)}{\partial x_i \partial x_j}\bigg|_{x=x^*} \right]$ of the Lagrange function $L(x;\lambda)$ are positive. Therefore, by virtue of Sylvester's criterion [66], the Hessian matrix $H(L)$ is positive-definite. Consequently, in view of the fact that $d^2 L(x;\lambda)\big|_{x=x^*} = \sum_{i=1}^{M} \sum_{j=1}^{M} H(L)_{i,j} \, dx_i dx_j > 0$, we can state that $d^2 L(x;\lambda)\big|_{x=x^*} > 0$. Then, by virtue of the theorem of the second-order sufficient condition for a local extremum [67], the found stationary point $x^* \in \mathbb{R}_+^M$ with coordinates (8) is a point of local minimum for the Lagrange function

$L(x;\lambda)$ and, consequently, this point is a local minimum for the considered original optimization problem (5).

Now let us find out whether the found local minimum $x^* \in \Omega$ of the optimization problem (5) is also its global minimum. For this, we note that the objective function $\sigma^2(x)$ of problem (5), if we consider it as a quadratic form, is a canonical form with positive coefficients, and, therefore, it is a positive definite quadratic form at $\forall x \in \mathbb{R}^M/\{0\}$. Then, by virtue of the well-known theorem [67], the quadratic form $\sigma^2(x)$ is an infinitely growing function on any closed set $X \subset \mathbb{R}^M$, in particular, on our feasible set Ω. We need the following fact [67], [68]: if any function $f(x)$ is a continuous and infinitely growing function on a set $X \subset \mathbb{R}^n$, then a global solution to the problem $\underset{x \in X}{\text{minimize}}(f(x))$ exists. Since our function $\sigma^2(x)$, which, as it has already been shown, is an infinitely increasing function, is also a continuous function, then, by virtue of the above fact, $\sigma^2(x)$ has a global minimum at Ω. In other words, optimization problem (5) has a global solution. On the other hand, the point $x^* \in \mathbb{R}^M_+$ with coordinates (8) is the only stationary point of the function $\sigma^2(x)$. Hence, this point is also that global solution to the optimization problem (5), the existence of which has just been proved.

Remark 3. To avoid possible misunderstandings among readers inexperienced in mathematics who are interested in the agricultural subject matter of the present paper, just in case, we will briefly explain the essence of an infinitely growing function (this is not the same as an upper-unbounded function!). Definition: a function $f(x)$ defined on a set $X \subseteq \mathbb{R}^n$ is called an infinitely growing on X if $\underset{k \to \infty}{\lim\sup} f(x^{\{k\}}) = \infty$ [69] for any sequence $\{x^{\{k\}}\}_{k \in \mathbb{N}} \subset X$ such that either $\underset{k \to \infty}{\lim} x^{\{k\}} = x \in \overline{X} \setminus X$ or $\underset{k \to \infty}{\lim} \|x^{\{k\}}\| = \infty$. Let us explain the essence of this definition using two examples. Consider a function $f(x) = x_1^2 - x_1 x_2 + x_2^2$ on a set $X = \mathbb{R}^2$, and show that this function is an infinitely growing function in $\mathbb{R}^2$. Indeed, using inequality $x_1 x_2 \le \dfrac{x_1^2 + x_2^2}{2}$, we can write: $f(x) \ge \dfrac{1}{2}(x_1^2 + x_2^2) = \dfrac{1}{2}\|x\|^2$, where $x = (x_1, x_2)^T$ is a column vector, $\|x\|$ means Euclidean norm. Therefore, for any sequence $\{x^{\{k\}}\}_{k \in \mathbb{N}} \subset \mathbb{R}^2 : \underset{k \to \infty}{\lim} \|x^{\{k\}}\| = \infty$, we obtain that $f(x^{\{k\}}) \xrightarrow{k \to \infty} \infty$. Another example: it is easy to verify that the function $f(x) = x_1^2 + 4x_1 x_2 + x_2^2$ (which differs from the function of the previous example only by the

coefficient at the term $x_1 x_2$) is not an infinitely growing function on $X = \mathbb{R}^2$. Indeed, since $f(x) = (x_1 + 2x_2)^2 - 3x_2^2$, taking any such sequence $\{x^{\{k\}}\}_{k \in \mathbb{N}} \subset \mathbb{R}^2 : \underset{k \to \infty}{\lim} \|x^{\{k\}}\| = \infty$, so that the first term in the expression $f(x^{\{k\}})$ turns to zero, for example, the sequence $x^{\{k\}} = (-2x_2^{\{k\}}, x_2^{\{k\}})^T$, where $x_2^{\{k\}} \xrightarrow{k \to \infty} \infty$ (for example, $x_2^{\{k\}} = k$), we get that $\underset{k \to \infty}{\lim\sup} f(x^{\{k\}}) = \underset{k \to \infty}{\lim\sup}\left(-3(x_2^{\{k\}})^2\right) = -\infty$, that is, the mandatory requirement $\underset{k \to \infty}{\lim\sup} f(x^{\{k\}}) = \infty$ in the definition of an infinite growing function is not met. Therefore, this function is not an infinitely growing function on the set $X = \mathbb{R}^2$. Finally, let us show that our objective function $\sigma^2(x)$ is an infinitely growing function on a feasible set Ω. As the reader remembers, we already proved this fact before Remark 2, but for the proof we used not the definition of an infinitely growing function, but other considerations. The somewhat unusual proof below, directly using the definition of an infinitely growing function, was kindly provided to us by our colleague Ruslans Aleksejevs from the Faculty of Mechanics and Mathematics, Lomonosov Moscow State University (see Acknowledgments). Statement (R. Aleksejevs): if the set $X \subset \mathbb{R}^M$ is compact (in $\mathbb{R}^M$ this is equivalent to the fact that the set X is bounded and closed), then any function $f(x)$ on X is an infinitely growing function. We carry out the proof of this statement by contradiction of the rule of contraries: suppose that there is a sequence $\{x^{\{k\}}\}_{k \in \mathbb{N}} \subset X$ such that either

– type I: $\underset{k \to \infty}{\lim} x^{\{k\}} = x \in \overline{X} \setminus X$, or

– type II: $\underset{k \to \infty}{\lim} \|x^{\{k\}}\| = \infty$,

but in both types the function $f(x)$ is not an infinitely growing function, i.e. in both types of sequence $\{x^{\{k\}}\}_{k \in \mathbb{N}} \subset X$ there is valid $\underset{k \to \infty}{\lim\sup} \|f(x^{\{k\}})\| \ne \infty$. Since the set X is closed, then $\overline{X} \setminus X = \varnothing$ which means that type I is eliminated, that is, in X such sequences cannot exist. Therefore, the sequence $\{x^{\{k\}}\}_{k \in \mathbb{N}} \subset X$, that we assume to exist can only be of type II. However, the set X is also limited and, therefore, any sequence from X must be limited, i.e. there is such a constant $Const < \infty$, that $\|x^{\{k\}}\| \le Const$ for $\forall k \in \mathbb{N}$ [69]. Therefore, type II also is eliminated, because equality $\underset{k \to \infty}{\lim} \|x^{\{k\}}\| = \infty$ means that the

sequence $\left\{x^{\{k\}}\right\}_{k \in \mathbb{N}}$ is an infinitely large sequence, and any infinitely large sequence is unbounded. Thus, we got a contradiction. In other words, we have proved that our assumption that on a compact set $X \subset \mathbb{R}^M$ the function $f(x)$ is not an infinitely growing function turned out to be wrong. The statement is proven. Since our feasible set Ω is a closed simplex and, therefore, a compact set, it immediately follows from the proved statement: our objective function $\sigma^2(x)$ is an infinitely growing function on Ω. EOR

So, we got that the point $x^* \in \mathbb{R}_+^M$, whose coordinates are calculated by formula (8) is a global solution to the optimization problem (5). Taking into account (8) in (2) and (4) gives us the following results, respectively:

$$R^* \overset{def}{\equiv} R\left(x^*\right) = \frac{S}{\sum_{j=1}^{M} \frac{1}{p_j^2 \sigma_j^2}} \sum_{i:\, p_i m_i > c_i} \frac{p_i m_i - c_i}{p_i^2 \sigma_i^2}$$
$$+ \sum_{i:\, p_i m_i \le c_i} \frac{\left(\hat{p}_i m_i - c_i\right) Q_i}{m_i} + \sum_{i:\, p_i m_i > c_i} \left(\hat{p}_i - p_i\right) Q_i; \quad (9)$$

$$\left(\sigma^2\right)^* \overset{def}{\equiv} \sigma^2\left(x^*\right) = \frac{S^2}{\left(\sum_{i=1}^{M} \frac{1}{p_i^2 \sigma_i^2}\right)^2} \sum_{i:\, p_i m_i > c_i} \frac{1}{p_i^2 \sigma_i^2}$$
$$+ \sum_{i:\, p_i m_i \le c_i} \frac{p_i^2 \sigma_i^2 Q_i^2}{m_i^2}. \quad (10)$$

It is easy to see that if $p_i m_i - c_i > 0$ for $\forall i = \overline{1, M}$, then formulas (9) and (10) take a more compact form:

$$R^* = \frac{S}{\sum_{j=1}^{M} \frac{1}{p_j^2 \sigma_j^2}} \sum_{i=1}^{M} \frac{p_i m_i - c_i}{p_i^2 \sigma_i^2} + \sum_{i=1}^{M} \left(\hat{p}_i - p_i\right) Q_i;$$

$$\left(\sigma^2\right)^* = \frac{S^2}{\sum_{i=1}^{M} \frac{1}{p_i^2 \sigma_i^2}}.$$

Let us interpret the obtained formulas (8)-(10) aggregate. If an agricultural enterprise will be seeding an i-th kind of agricultural crop on an x_i^* ha area (according to the calculation formula (8)) from the available cultivation field with an area of S ha, then the total profit of the enterprise from the sale of the harvested crop while fulfilling the terms of the procurement contract in full scope will be R^* euros (formula (9)), which is guaranteed in the sense that it is achieved with the least risks $\left(\sigma^2\right)^*$ (formula (10)) from a possible loss of yield of cultivated M crop species associated with possible changes in hydro-meteorological conditions.

Along with formulas (8)-(10), the following calculation formulas are also useful from a practical point of view:

– the volume (quintal) of the harvested crop of the i-th $\left(i = \overline{1, M}\right)$ kind of agricultural crop species:

$$m_i x_i^* = \begin{cases} \dfrac{Sm_i}{p_i^2 \sigma_i^2 \sum_{j=1}^{M} \frac{1}{p_j^2 \sigma_j^2}} & if \quad p_i m_i - c_i > 0, \\ Q_i & otherwise; \end{cases} \quad (11)$$

– the total volume (quintal) of the obtained harvest of all M kinds of agricultural crop species:

$$\sum_{i=1}^{M} m_i x_i^* = \sum_{i:\, p_i m_i \le c_i} Q_i + \frac{S}{\sum_{j=1}^{M} \frac{1}{p_j^2 \sigma_j^2}} \sum_{i:\, p_i m_i > c_i} \frac{m_i}{p_i^2 \sigma_i^2}. \quad (12)$$

Finally, let us note that if, for some reason, we are not interested in net results $\left(c_i > 0, \forall i = \overline{1, M}\right)$ (for example, we do not have the corresponding necessary source data), but only in gross results $\left(c_i = 0, \forall i = \overline{1, M}\right)$, i.e. results without taking into account the total costs of an agricultural enterprise at all stages of the cultivation process in the sowing field of all M kinds of agricultural crops, then all calculation formulas (8)-(12) will be absolutely valid for this case: it is just needed to take $c_i = 0, \forall i = \overline{1, M}$ in these formulas, resulting in a more compact form (except for formula (10) – it does not change). For the convenience of use for any persons associated with the cultivation of crops, these calculation formulas are given below:

$$x_i^* = \frac{S}{p_i^2 \sigma_i^2 \sum_{j=1}^{M} \frac{1}{p_j^2 \sigma_j^2}} > 0, \forall i = \overline{1, M};$$

$$R^* = \frac{S}{\sum_{j=1}^{M} \frac{1}{p_j^2 \sigma_j^2}} \sum_{i=1}^{M} \frac{m_i}{p_i \sigma_i^2} + \sum_{i=1}^{M} \left(\hat{p}_i - p_i\right) Q_i;$$

$$m_i x_i^* = \frac{Sm_i}{p_i^2 \sigma_i^2 \sum_{j=1}^{M} \frac{1}{p_j^2 \sigma_j^2}}, \forall i = \overline{1, M};$$

$$\sum_{i=1}^{M} m_i x_i^* = \frac{S}{\sum_{j=1}^{M} \frac{1}{p_j^2 \sigma_j^2}} \sum_{i=1}^{M} \frac{m_i}{p_i^2 \sigma_i^2}.$$

Let us emphasize once again that the total profit R^* of an agricultural enterprise, determined by formula (9), is not the result of solving either the problem of finding the maximum profit, or the problem of finding the maximum harvest, and is the result of solving the problem of minimizing risks from possible losses in yield capacity due to changes in hydro-meteorological conditions. In other

words, the total profit R^* of an agricultural enterprise, which is calculated according to formula (9), is the result of the implementation of the optimal plan for seeding all M kinds of agricultural crops strictly according to the law (8), and this law, in its turn, is the result of the attaining of a single goal – to ensure that the financial/monetary risks from possible losses in crop yields are kept at minimum. In short, the profit function (2) was driven by the risk function (i.e. by objective function) $\sigma^2(x)$ on the grounds of $\sigma^2(x) \to \min$. There could arise a natural question: how much will the obtained results change if, in the study of the same problem, one proceeds not from the consideration of ensuring the minimum risks, but from the consideration of ensuring the maximum of the obtained profit, or from the consideration of ensuring the maximum harvest? The answer to this question will be discussed in the next section.

III. MATHEMATICAL MODEL OF THE CONSIDERED PROBLEM FROM THE ENSURING MAXIMUM PROFIT OR MAXIMUN HARVEST STANDPOINT

This section considers the same agricultural problem that was formulated and formalized in the previous section, but now this problem will be investigated from the point of view of either maximizing the profit from the sale of the harvested crop (goal (B)), or maximizing the harvest of cultivated crop species (target (C)). Let us note that the notation used in this section has the same meaning as in the previous section.

If we consider the agrarian problem from the point of view of the goal (B), then we get the following model, which is a one-criterion linear programming problem:

$$\underset{x \in \Omega}{\text{maximize}}\left[R(x) = \sum_{i=1}^{M}(p_i m_i - c_i)x_i \right.$$
$$\left. + \sum_{i=1}^{M}(\hat{p}_i - p_i)Q_i \right], \tag{11}$$

where

$$\Omega = \left\{ x \in \mathbb{R}_+^M : \sum_{i=1}^{M} x_i = S, \, m_i x_i \geq Q_i, \, \forall i = \overline{1,M} \right\}.$$

If we consider the agrarian problem from the point of view of the goal (C), then we get the following one-criterion linear programming problem:

$$\underset{x,V \in \Theta}{\text{maximize}}[V], \tag{12}$$

where

$$\Theta \overset{def}{\equiv} \left\{ x \in \mathbb{R}_+^M : \sum_{i=1}^{M} x_i = S, \, \sum_{i=1}^{M} m_i x_i - V \geq 0, \right.$$
$$\left. m_i x_i \geq Q_i, \, \forall i = \overline{1,M} \right\}.$$

In model (12), V is a new variable meaning the unknown cumulative guaranteed harvest of all M kinds of agricultural crops grown that we want to maximize.

Having solved problems (11) and (12) by some analytical or numerical method, for example, simplex-method, generalized reduced gradient method, etc., we find the desired optimal plans $x^* = \underset{x \in \Omega}{\arg\max}\, R(x)$ and $\left(x^*, V\right)^* = \underset{x,V \in \Theta}{\arg\max}[V]$, respectively. Obviously, the found optimal plan $x^* = \underset{x \in \Omega}{\arg\max}\, R(x)$ also makes it possible to answer the question about the volume (quintal) $m_i x_i^*$ of the harvested crop of an i-th $\left(i = \overline{1,M}\right)$ kind of agricultural crop species and, consequently, the question about the total volume (quintal) $\sum_{i=1}^{M} m_i x_i^*$ of the harvested crop of all M kinds of agricultural crops; the found optimal plan $\left(x^*, V\right)^* = \underset{x,V \in \Theta}{\arg\max}[V]$ also makes it possible to answer both the question about the profit $R\left(x^*\right)$ gained by an agricultural enterprise, as well as the question about the volume (quintal) $m_i x_i^*$ of the harvested crop of an i-th $\left(i = \overline{1,M}\right)$ kind of agricultural crop species.

IV. RESULTS AND DISCUSSION

A. Numerical Experiment: Implementation of Mathematical Models

In this section, using a specific example, the application of the developed mathematical models (5), (11) and (12) outlined in the previous two sections is illustrated. As an example, we consider the cultivation of common buckwheat, leguminous crops, grain maize, common oat, common millet, bread wheat, bread rye and common barley in the Orel region of the Russian Federation for the time period 2011-2020. All the necessary source data are real data that we have extracted both from open sources [70]-[76], from the source [77] requiring paid access, as well as through the personal communication channels of the first and third co-authors of this work with experts and competence centers in Orel city. The collected source data are averaged over years and by territorial units, real data (i.e. they are data for the Orel region as a whole for each year of the considered period 2011-2020) of the following kinds: (a) yield (quintal/ha) for each of the abovementioned 8 kinds of crops; (b) cultivated area (ha) allocated for each kind of crop; (c) the market price (RUB/quintal) for each crop species; (d) total costs (RUB/ha) for all stages of cultivation of each kind of crop (except for 2020; see Remark 4); (e) the volume (quintal) of the crop of each kind, which was sold to the procurer at the purchase price in accordance with the terms of the concluded procurement contracts; (f) the volume (quintal) of each crop harvested.

Remark 4. Note that we could not get reliable data on the actual net profit of agricultural enterprises in the Orel region gained from the sale of the total harvest of all 8 kinds of crops for the studied period 2011-2020. However, using statistical data of kinds (a)-(f), it was easy to calculate that the total harvest of all 8 kinds of crops grown on sown

fields with a total area of 8530450 ha amounted to 294148550 quintals, from the sale of which the total net profit of agricultural enterprises in the Orel region for the period 2011-2020, taking into account the fulfilment of the conditions of existing procurement contracts, amounted to 4,947,644,993 roubles. It should also be noted that we could not get reliable information on the total costs (RUB/ha) for all stages of growing all 8 kinds of crops for 2020. However, having calculated the trends in the dynamics of total costs for previous years, using the analytical alignment method applying a polynomial of the second degree (parabolic type), we extrapolated the available and obtained data for the year 2020. EOR

Let us get back to the collected source real data and note the following important circumstance. Before using the available source data (except for data types (b) and (f), since these data are not regarded to be the necessary source data for mathematical models (5), (11), (12), on the contrary, for these mathematical models the data types (b) and (f) are the required unknowns) for the particular example under consideration by applying mathematical models (5), (11), (12), we preliminarily carried out, using the Kruskal's minimum spanning tree algorithm [78], hierarchical clustering of yields (quintal/ha) of agricultural crops of 8 species by year with respect to the following hydro-meteorological signs (daily data for the Orel region during 2011-2020): precipitation, air humidity, air temperature, soil temperature, wind speed, solar activity [79], [80]. As a result, we got 4 clusters: a cluster of yields in 2011, 2014, 2015; cluster of yields in 2012, 2017; cluster of yields in 2013, 2016, 2018; cluster of yields in 2019, 2020. Then, carrying out statistical processing of the source data of the (a), (c)-(e) types corresponding to the years of each of the four clusters, all processed source data of (a), (c)-(e) types were distributed over four clusters (obviously, with the same years), for each of which the radius, diameter and centers were calculated. Finally, using the results obtained, we determined: conditional year; conditional yield (quintal/ha) for each of the 8 kinds of crops: $\{m_i\}=\{12.26; 22.41; 68.16; 23.92; 17.85; 37.99; 30.44; 32.07\}$; conditional market price (RUB/quintal) for the crop of each kind: $\{p_i\}=\{1,551.71; 1,856.59; 713.58; 644.88; 758.02; 765.26; 601.35; 822.65\}$; conditional total costs (RUB/ha) for all stages of cultivation lifecycle for each kind of crops: $\{c_i\}=\{8,739.1; 19,029.6; 33,767.2; 14,705.2; 15,583.1; 25,276.2; 11,813.8; 18,850.9\}$; conditional volume (quintal) for each kind of harvested crop species for sale at the purchase price in accordance with procurement contract: $\{Q_i\}=\{161,300; 165,100; 473,850; 54,400; 900; 4,400,000; 24,750; 1,200,000\}$; conditional purchase price: $\{\hat{p}_i\}=\{1,396.54; 1,670.93; 642.22; 580.39; 682.22; 688.73; 541.21; 740.38\}$; biased sample variance of conditional yields: $\{\sigma_i^2\}=\{8.12; 13.98; 80.97; 12.79; 24.97; 67.49; 91.98; 32.04\}$. In other words, we have reduced the original example to an example in which one conditional agricultural enterprise in the Orel region, which owns one conditional plough land with an area of 853,045 ha (= 8,530,450/10, where 10 is the duration of the considered period 2001-2020), intends to seed this field with 8 kinds of agricultural crops in a conditional year. It is required to determine which kinds of crop species and in what proportions an agricultural enterprise should be seeding in order to:

- to harvest a crop with the least financial risks from a possible loss of crop yields associated with possible changes in hydro-meteorological conditions (i.e. strategy (A) is chosen);

- or the net profit from the sale of the harvested crop would be maximum (i.e. strategy (B) is chosen);

- or the total harvest would reach its maximum (i.e. strategy (C) is chosen).

It should be taken into account that an agricultural enterprise is obliged to comply with the terms of the existing procurement contract, regardless of which strategy of the three strategies listed above it will select.

If an agricultural enterprise chooses strategy (A), then a mathematical model (5) is selected, the implementation of which gives the following results (annual):

- Optimal seeding plan (ha): buckwheat: 27,755; leguminous: 19,388; maize: 131,242; oat: 160,694; millet: 50; wheat: 114,114; rye: 184,800; barley: 98,748.

- The resulting harvest volume (quintal): buckwheat: 340,272.56; leguminous: 434,477.62; maize: 8,945,442.2; oat: 3,843,801.3; millet: 900; wheat: 4,335,197; rye: 5,625,324.6; barley: 3,166,841.5.

- Total harvest volume (quintal): 26,692,256.78.

- Net profit (RUB): 3,458,774,455.96.

If an agricultural enterprise chooses strategy (B), then a mathematical model (11) is selected, the implementation of which gives the following results (annual):

- Optimal seeding plan (ha): buckwheat: 13,156.61; leguminous: 676,560.5; maize: 6,952.02; oat: 2,274.25; millet: 50.42; wheat: 115,819.95; rye: 813.07; barley: 37,418.15.

- The resulting harvest volume (quintal): buckwheat: 161,300; leguminous: 15,161,721.38; maize: 473,850; oat: 54,400; millet: 900; wheat: 4,400,000; rye: 24,750; barley: 1,200,000.

- Total harvest volume (quintal): 15,851,271.38.

- Net profit (RUB): 15,711,408,612.

Finally, if an agricultural enterprise chooses strategy (C), then a mathematical model (12) is selected, the implementation of which gives the following results (annual):

- Optimal seeding plan (ha): buckwheat: 13,156.61; leguminous: 7,367.25; maize: 6,761,453; oat: 2,274.25; millet: 50.42; wheat: 115,819.95; rye: 813.07; barley: 37,418.15.

- The resulting harvest volume (quintal): buckwheat: 161,300; leguminous: 165100; maize: 46,086,063.88; oat: 54,400; millet: 900; wheat: 4,400,000; rye: 24,750; barley: 1,200,000.

- Total harvest volume (quintal): 52,092,513.88.

- Net profit (RUB): 10,554,492,045.

Note that if we average over the years the areas where agricultural crops were actually grown in the Orel region, and the harvested crops, we get that

- Completed seeding "plan" (ha): buckwheat: 72,354; leguminous: 50,772; maize: 50,788; oat: 23,358; millet: 564; wheat: 463,348; rye: 4,754; barley: 187,107.

- The resulting harvest volume (quintal): 806,388; leguminous: 1,100,728; maize: 3,158,961; oat: 544,375; millet: 8,946; wheat: 17,712,353; rye: 123,716; barley: 5,959,388.

- Total harvest volume (quintal): 29,414,855.

- Net profit (RUB): 4,947,644,993.

B. Comparative Analysis of the Obtained Numerical Results

Let us compare the harvest volume $H(\text{Model})$ and profit $R(\text{Model})$ of each model with the actual harvest volume $H(\text{Real})$ and profit $R(\text{Real})$, respectively:

$$\begin{cases} H(5) - H(\text{Real}) = -2,722,598.22, \\ R(5) - R(\text{Real}) = -1,488,870,537.04; \end{cases}$$

$$\begin{cases} H(11) - H(\text{Real}) = -13,563,583.62, \\ R(11) - R(\text{Real}) = +10,763,763,619; \end{cases}$$

$$\begin{cases} H(5) - H(\text{Real}) = +22,677,658.88, \\ R(12) - R(\text{Real}) = +5,606,847,052. \end{cases}$$

Let us interpret the obtained comparison results:

- Application of model (5), the essence of which is a cautious strategy, gives ≈9% and ≈30% less harvested crop volume and profit, respectively, in comparison with the actually obtained harvest and profit;

- Application of model (11), the purpose of which is to ensure the maximum profit, gives ≈46% less harvested crop, but increases the profit ≈3.2 times compared to the real situation;

- Application of the model (12), the purpose of which is to obtain the maximum harvested crop, gives ≈1.77 and ≈2.3 times more harvested crop and profit, respectively, in comparison with the actually obtained harvest and profit.

Despite the foregoing, it must be taken into account that the projection of a multidimensional system to which the studied example belongs (different years; different agricultural enterprises with different technical, financial and other resource capabilities; different cultivated fields with different agrochemical, agrophysical characteristics, etc.), onto a one-dimensional system (one year; one agricultural enterprise; one seeding field, equally suitable for growing 8 kinds of field crops; etc.), and then groundless extrapolation of the results of solving a one-dimensional system to a multidimensional system may lead to false conclusions, much different from the true situation, to overly rosy or, conversely, to overly pessimistic forecasts. In the context of the agricultural problem we are considering, at least it should be borne in mind that for the main agrophysical, agrochemical, landscape-ecological, ecological-genetic, biological, etc. characteristics of agricultural land in the Orel region (over 2030 thousand ha, of which the area of plough-land is about 74% [73], [81]-[88]) are not homogeneous – there are more than 240 soil varieties: over 17 thousand ha of agricultural land are subject to water erosion, and about 21% of them are moderately – and strongly eroded; in terms of qualitative composition, about 8% of plough-land has sod-podzol and pale-gray forest soils with a low level of cultivation, which are characterized by high acidity (4÷4.5 pH), low content of organic matter (1÷2.5 %), insufficient amount of mineral nutrients such as nitrogen, phosphorus, potassium, magnesium and calcium; etc. [81], [86]-[88]. All these circumstances must be taken into account. However, in this section, while applying mathematical models (5), (11), (12) to the considered real example, these circumstances were not considered: the reason for this is not the limitations of the proposed mathematical models, not the impossibility or complexity of the mechanism for taking into account the above listed circumstances, and the reason is only that the authors of this work did not have access to reliable information about agrophysical, agrochemical, landscape-ecological, ecological-genetic, biological, etc. characteristics of plough-lands in the Orel region, on which in 2011-2020 the kinds of crop species we are studying were grown.

In conclusion, the author would like to emphasize that it is possible to combine all these three mathematical models (more precisely, three different strategies of these models) in a compromise way into one: then we get one multicriteria optimization model, the solution of which will give us a Pareto-optimal plan, i.e. we get a trade-off decision.

V. CONCLUSIONS

This paper investigates the problem of quantitative control of the seeding of the plough-land by agricultural crops in various agro-climatic conditions. The considered problem is studied from the perspective of three strategies: from the seeding planning perspective aiming at minimal risk associated with possible unfavourable agroclimatic conditions (a probabilistic approach is used); from the perspective of obtaining the maximum crops sales profit (a

deterministic approach is used); from the perspective of obtaining the maximum crops harvest. In this work, mathematical models are constructed for each strategy and their respective solutions are found. In addition, in this work, a specific real example is considered, which illustrates the application of the constructed mathematical models. Computer implementation of these models (using both Mathcad software and Microsoft Excel spreadsheet) makes it possible to find the optimal seeding plan for the crops under consideration for each of the three strategies (A)-(C). Finally, a comparative analysis of the found plans of the best seeding is carried out.

REFERENCES

[1] Latvian Environment, Geology and Meteorology Centre, "Latvijas klimats." [Online]. Available: https://www.meteo.lv/lapas/laiks/latvijas-klimats/latvijas-klimats?id=1199&nid=562 [Accessed: March 14, 2021]

[2] D. Spaar, H. Kleinhempel, and R. Fritzsche, Getreide, mais und futtergräser. Berlin, Germany: Springer-Verlag, 2013.

[3] J. S. Rayfield, K. L. Smith, T. D. Park, and D. B. Croom, Principles of agriculture, food, and natural resources: Applied Agriscience. Tinley Park: Goodheart-Wilcox Publisher, 2015.

[4] S. Clark, Ed., Sustainable agriculture-beyond organic farming. Basel: MDPI AG, 2016.

[5] A. N. Esaulko, V. V. Ageev, and L. S. Gorbatko, Agrochemical survey and monitoring of soil fertility. Stavropol: Stavropol State Agrarian University, 2013.

[6] R. E. Planant, Spatial data analysis in ecology and agriculture using R. Boca Raton: CRC Press, 2018.

[7] A. V. Kilchevsky and L. V. Khotyleva, Eds., Genetic foundations of plant breeding: General plants genetics. Minsk: Belarusian Science, 2008.

[8] A. V. Kilchevsky and L. V. Khotyleva, Eds., Genetic foundations of plant breeding: Private plants genetics. Minsk: Belarusian Science, 2010.

[9] A. N. Yadav, J. Singh, A. A. Rastegari, and N. Yadav, Eds., Plant microbiomes for sustainable agriculture. Cham: Springer Nature, 2020.

[10] E. I. Koshkin and G. G. Guseinov, Ecological physiology of agricultural crops. Moscow: Prospect Publishing, 2020.

[11] A. Nurbekov, A. Kassam, D. Sydyk, Z. Ziyadullaev, I. Jumshudov, H. Muminjanov, D. Feindel, and J. Turok, Practice of conservation agriculture in Azerbaijan, Kazakhstan and Uzbekistan. Ankara, Turkey: Food and Agriculture Organization of the United Nations Publishing, 2016.

[12] A. Nurbekov, A. Musaev, D. Sydyk, Z. Ziyadullaev, and J. Turok. Conservation agriculture in irrigated areas of Azerbaijan, Kazakhstan and Uzbekistan. Beirut: International Centre for Agricultural Research in the Dry Areas, 2015.

[13] Ch. Gienapp, D. Dreger, A. Zaharenko, etc., Cereal crops: growing, harvesting, reworking and using. Moscow: "DLV AgroDelo" Publishing, 2008.

[14] "European Commission. EU prices for selected representative products. Monthly prices for vegetal products." [Online]. Available: https://ec.europa.eu/info/sites/info/files/food-farming-fisheries/farming/documents/market-prices-vegetal-products_en.pdf [Accessed: March 14, 2021]

[15] Ministry of Agriculture of the Republic of Latvia, "Agriculture Annual Reports. Agricultural Report 2019 for 2018." [Online]. Available: https://www.zm.gov.lv/public/files/CMS_Static_Page_Doc/00/00/01/62/36/2019_lauks_gada_zinojums.pdf [Accessed: March 14, 2021]

[16] Latvian Rural Advisory and Training Centre. [Online]. Available: http://new.llkc.lv/ [Accessed: March 14, 2021]

[17] Food and Agriculture Organization of the United Nation, "FAOSTAT: Crops." [Online]. Available: http://www.fao.org/faostat/en/#data/QC [Accessed: March 14, 2021]

[18] Department of Primary Industries and Regional Development. Goverment of Western Australia, "Agriculture and Food: Sowing." [Online]. Available: https://www.agric.wa.gov.au/crops/production-postharvest/sowing [Accessed: March 14, 2021

[19] O. Diekman, H. Heesterbeek, and T. Britton, Mathematical tools for understanding infectious disease dynamics. Princeton: Princeton University Press, 2012.

[20] J. W. Jones, J. M. Antle, B. Basso, et al, "Toward a new generation of agricultural system data, models, and knowledge products: State of agricultural systems science," Agricultural Systems, vol. 155, pp. 269-288, 2017.

[21] J. H. M. Thornley and J. France, Mathematical models in agriculture: quantitative methods for the plant, animal and ecological sciences. Wallingford: CABI Publishing, 2007.

[22] M. A. Vodounon, "Mathematical models can make farmers more efficient," African Technology Forum, vol. 8, no. 1, 5 p., 2017.

[23] P. Mérel and R. E. Howitt, "Theory and application of positive mathematical programming in agriculture and the environment," Annual Review of Resource Economics, vol. 6, pp. 451-470, 2014.

[24] V. A. Pavljushin and N. N. Semenova, Eds., Mathematical modelling in plant protection. St. Petersburg: All-Russian Research Institute of Plant Protection Press, 2014.

[25] Ch. B. S. Teh, Introduction to mathematical modelling of crop growth. Boca Raton: BrownWalker Press, 2006.

[26] P. K. Thornton and G. Hoogenboom, "A computer program to analyze single-season crop model outputs," Agronomy Journal, vol. 86, no. 5, pp. 860-868, 1994.

[27] C. S. Bernard and J. S. Nix, Farm planning and control. Cambridge: Cambridge University Press, 1980.

[28] P. B. R. Hazell, and R. D. Norton, Mathematical programming for economic analysis in agriculture. London: Macmillan Publishing, 1986.

[29] J. Glen, "Mathematical models in farm planning: A survey," Operations Research, vol. 35, pp. 641-666, 1987.

[30] J. France and J. H. M. Thornley, Mathematical models in agriculture: A quantitative approach to problems in agriculture and related sciences. Boston: Butterworths Publishing, 1984.

[31] R. Beneke and R. Winterboer, Linear programming: Application to agriculture. Ames: Iowa State University Press, 1973.

[32] G. E. Dalton, Ed., Study of agricultural systems. London: Applied Science Publishing, 1975.

[33] V. D. Baranov and I. G. Tarakanov, Crops harvest programming. Moscow: Patrice Lumumba Peoples' Friendship University Press, 1990.

[34] A. N. Polevoy. Theory and calculation of the productivity of agricultural crops. Leningrad: GidroMeteoIzdat Publishing, 1988.

[35] A. N. Polevoy. Applied modelling and forecasting of crops productivity. Leningrad: GidroMeteoIzdat Publishing, 1988.

[36] M. K. Kayumov, Programming the productivity of field crops. Moscow: RosAgroPromIzdat Publishing, 1989.

[37] N. I. Mozhaev, N. A. Serikpaev, and G. Zh. Stybaev, Crops harvest programming. Astana: Foliant Publishing, 2013.

[38] V. V. Gritsenko and V.E.Dolgodvorov, Fundamentals of programming crop yields. Moscow: AgroPromIzdat Publishing, 1986.

[39] S. A. Vorobiev, A. N. Kartashev, A. M. Lykov, and I. P. Makarov, Arable farming. Moscow: AgroPromIzdat Publishing, 1991.

[40] J. Baier, V. Cerny, M. Ferik, et al., Harvest formation of main agricultural crops. Moscow: KOLOS Publishing, 1984.

[41] V. A. Dukhovny, S. A. Nerozin, G. V. Stulina, and G. F. Solodkiy, Crops harvest programming: System approach into applied to amelioration. Tashkent: 2015.

[42] M. G. Kendall and A. Stuart, The advanced theory of statistics: Inference and relationship. London: Charles Griffin & Company, Ltd., 1961.

[43] D. R. Cox and E.J.Snell, Applied statistics: Principles and examples. Boca Raton: CRC Press, 2020.

[44] A. S. C. Ehrenberg, Data reduction: Analysis and interpretation of statistical data. New York: John Wiley & Sons, 1975.

[45] A. S. C. Ehrenberg, A Primer in Data Reduction: An Introductory Statistics Textbook. New York: John Wiley & Sons, 2007.

[46] A. N. Shiryaev, Probability. New York: Springer-Verlag, 1996.

[47] B. V. Gnedenko and I. A. Ushakov, Theory of Probability. Boca Raton: CRC Press, 1988.

[48] A. A. Samarskii and A. P. Mikhailov, Principles of mathematical modelling: Ideas, methods, examples. Boca Raton: CRC Press, 2018.

[49] N. N. Moiseev, Selected works: Interdisciplinary studies of global problems. Moscow: Tydex Co, 2003.

[50] N. N. Moiseev, Numerical methods in the theory of optimal systems. Moscow: Nauka Publishing, 1971.

[51] N. N. Moiseev, Mathematical problems of system analysis. Moscow: Nauka Publishing, 1981.

[52] A. N. Bogolyubov. Fundamentals of mathematical modelling. Moscow: Moscow State University Press, 2001.

[53] O. G. Alekseev, Complex application of discrete optimization methods. Moscow: Nauka Publishing, 1987.

[54] B. Ya. Sovetov and S. A. Yakovlev. Modelling of systems. Moscow: Higher School Publishing, 2001.

[55] S. V. Avrutik, Ed., Solving nonlinear programming problems in a deterministic, discrete, and probabilistic setting: Algorithms and programs. Irkutsk: Siberian Power Engineering Institute of the Siberian Branch of the USSR Academy of Sciences Press, 1976.

[56] I. B. Zagaytov and L. P. Yanovsky, Prediction, foresight, forecast in reproduction planning. Voronezh: Voronezh State Agrarian University Press, 2011.

[57] I. B. Zagaytov, L. P. Yanovsky, V. G. Raskin, and S. N. Dementyev, The 'ZONT' method in forecasting fluctuations of natural harvest conditions. Voronezh: Voronezh State Agrarian University Press, 1996.

[58] I. B. Zagaytov and L. S. Vorobyova, Forecast of fluctuations of the natural conditions of agricultural production, and world statistics of harvests. Voronezh: Voronezh State Agrarian University Press, 1998.

[59] E. V. Zakshevskaya, S. V. Kuksin, and I. B. Zagaytov, "The strategy of state regulation of the Russian grain market based on the grain production forecasts," Herald of the Voronezh State Agrarian University, vol. 4, no. 51, pp. 169-178, 2016.

[60] I. B. Zagaytov, Planning and forecasting the development of agricultural-industrial complex. Voronezh: Voronezh State Agrarian University Press, 2004.

[61] Yu. S. Khokhlov, "Stable distributions and their generalizations, structure and limit theorems," Dr.Sc.Math. thesis, Steklov Mathematical Institute of the Russian Academy of Sciences, Moscow, Russia, 1996.

[62] B. V. Gnedenko, Cours of probability thoery. Moscow: Editorial URSS, 2005.

[63] G. A. Holton, Value at risk: Theory and practice. San Diego: Academic Press, 2003.

[64] P. Lévy, "Propriétés asymptotiques des sommes de variables aléatoires indépendantes ou enchaînées," Journal de Mathématiques Pures et Appliquées, vol. 14, no. 4, pp. 347-402, 1935.

[65] H. Cramer, "Über eine eigenschaft der normalen verteilungsfunktion," Mathematische Zeitschrift, vol. 41, no. 1, pp. 405-414, 1936.

[66] R. A. Horn and Ch. R. Johnson, Matrix analysis. Cambridge: Cambridge University Press, 1985.

[67] F. P. Vasiliev, Methods for solving extremal problems. Moscow: Nauka, 1981.

[68] A. G. Sukharev, A. V. Timokhov, and V. V. Fedorov, A course of optimization methods. Moscow: Nauka, 1986.

[69] V. A. Ilyin, V. A. Sadovnichiy, and Bl. H. Sendov, Mathematical analysis: Basic course. Moscow: Lomonosov Moscow State University Press, 1985.

[70] The Unified Interdepartmental Statistical Information System (UniSIS), "Yield of agricultural crops." [Online]. Available: https://www.fedstat.ru/indicator/31533 [Accessed: March 25, 2021]

[71] The Unified Interdepartmental Statistical Information System (UniSIS), "Gross harvest of agricultural crops." [Online]. Available: https://www.fedstat.ru/indicator/30950 [Accessed: March 25, 2021]

[72] Federal State Statistics Service of the Russian Federation, "Official Statistics." [Online]. Available: https://rosstat.gov.ru/ [Accessed: March 25, 2021]

[73] K. E. Laikam, A. V. Bazarov, N. A. Vorobyova, et. al., Eds., Results of the 2016 All-Russian Agricultural Census, Vol. 1: Highlights, Book 2: The main results of for the constituent entities of the Russian Federation. Moscow: "Statistics of Russia" Information and Publishing Centre, 2018.

[74] T. P. Ustinova, N. E. Dmitrieva, L. I. Akimova, M. V. Andreyuk, and M. Yu. Balakhnev, Eds., Orel region in figures, 2011-2016: Brief statistical collection. Orel: Territorial body of the Federal State Statistics Service for the Orel Region, 2017.

[75] O. V. Sidorenko, "Efficiency of grain production in agricultural organizations: factors and priorities of ensuring," All-Russia Research and Trade Journal of Legumes and Groat Crops, vol. 4, no 36, pp. 236-14, 2020

[76] Department of Agriculture of the Orel Region, "Department activities." [Online]. Available: http://apk.orel-region.ru/index.php?cont=97 [Accessed: March 14, 2021]

[77] Expert and Analytical Center of Agribusiness, "AB-Center." [Online]. Available: https://ab-centre.ru/ [Accessed: March 25, 2021]

[78] J. B. Kruskal, "On the shortest spanning subtree of a graph and the traveling salesman problem," Proceedings of the American Mathematical Society, vol. 7, no 1, pp. 48-50, 1956.

[79] Hydro-meteorological Research Center of Russian Federation, "Actual weather archive." [Online]. Available: https://meteoinfo.ru/archive-pogoda/russia/oryol-area [Accessed: March 20, 2021]

[80] Research and Production Centre "Map Maker", "GIS Meteo: Weather diary in Orel region." [Online]. Available: https://www.gismeteo.ru/diary/4432/ [Accessed: March 20, 2021]

[81] T. L. Ivaneha and N. A. Studennikov, "The state of soils in the farms of the Orel region and State support measures of soil fertility," Bulletin of Rural Development and Social Policy: Theoretical and Practical Journal, vol. 4, no. 16, pp. 40-43, 2017.

[82] The Unified Interdepartmental Statistical Information System (UniSIS), "Sown area by agricultural crop kinds." [Online]. Available: https://www.fedstat.ru/indicator/37662 [Accessed: March 23, 2021]

[83] The Unified Interdepartmental Statistical Information System (UniSIS), "The structure of sown areas agricultural crop kinds." [Online]. Available: https://www.fedstat.ru/indicator/37661 [Accessed: March 23, 2021]

[84] K. E. Laikam, A. V. Bazarov, N. A. Vorobyova, et. al., Eds., Results of the 2016 All-Russian Agricultural Census, Vol. 4: Sown areas of agricultural crops and areas of perennial plantings and berry crops, Book 1: Areas of crops and perennial plantings. Moscow: "Statistics of Russia" Information and Publishing Centre, 2018.

[85] K. E. Laikam, A. V. Bazarov, N. A. Vorobyova, et. al., Eds., Results of the 2016 All-Russian Agricultural Census, Vol. 4: Sown area of agricultural crops and areas of perennial plantings and berry crops, Book 2: The structure of sown areas. Grouping of census objects according to the size of sown areas. Moscow: "Statistics of Russia" Information and Publishing Centre, 2018.

[86] N. K. Kruzhkov, V. T. Lobkov, and V. V. Napolov, Agroecological foundations of biologization of husbandry in the Central forest-steppe of the European part of Russia. Orel: Orel State Agrarian University Press, 2018.

[87] V. T. Lobkov, Using soil-biological factors in husbandry. Orel: Orel State Agrarian University Press, 2016.

[88] A. A. Tsvyrko, "High efficiency of the grain market in the post-crisis development of agribusiness," Russian Journal of Agricultural and Socio-Economic Sciences, vol. 11, no 11, pp. 1-8, 2012.