Environment. Technology. Resources. Rezekne, Latvia
Proceedings of the 13th International Scientific and Practical Conference. Volume 3, 54-57

Consequences of the Basic Law of Flat Interlocking of Involute Cylindrical Gears

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***Abstract* - Three consequences based on the basic law of the flat interlocking have been determined that allow the generation of asymmetric tooth profile on different poloidal circles. Gears with asymmetric profile are used in both mechanical engineering and measuring devices. Qualitative indicators of engagement that cannot be accomplished with a symmetric profile have been developed. A specific example of generation III has been presented. The qualitative indicators of engagement, which are invariant to the parameters of the tool, have been improved.**

Keywords - possibilities of generation, involute cylindrical gears, asymmetric profile

I. INTRODUCTION

The asymmetric profile of the teeth relates to the special gear transmissions in mechanical engineering for the following reasons:

- use of non-standard tool to produce the toothed wheels – by the method of centroid wrapping or copying in view of the lack of interference and clipping of the work piece [1] [4];
- a relatively small number of examples in literature for gears with asymmetric profile of the teeth [2] [3];
- a lack of systematic approach in considering the asymmetry of the tooth of the gear and the output contour [1] [2] [3] [4];
- adoption of asymmetric modification of the profile only for non-reversible gearings [2];
- adoption of asymmetric profile creates difficulties of computational nature in determining the parameters and the qualitative indicators of a gear transmission [3] [4];
- incomplete use of the accumulated stock of knowledge on the symmetrical profile of teeth that can with certain conditions be extrapolated onto the asymmetric profile.

II. EXPOSITION

Low-module gears are one of the main components in devices. The quality and reliability of these components determine the accurate and reliable operation of the whole device.

The specific requirements and conditions of operation have separated the small-module gears in a separate group for which the theoretical and experimental data obtained for large and medium module gears are not applicable.

The lack of sufficient information about the possibilities and accuracy in manufacturing gears with asymmetric tooth profile makes their implementation quite difficult.

In this regard, the investigation of small-module involute cylindrical gears with an asymmetric tooth profile is a pressing task of our industry.

Therefore, there is a need for development of a system for calculation of asymmetric profile without using special geometric parameters, which will allow for these gears to be widely used in mechanical and precision engineering [5] [6].

When the requirements for the gear cannot be satisfied by the symmetrical profile, asymmetric modification can be adopted and reversible or non-reversible gear with different from the hitherto known qualitative indicators can be obtained [7].

The basic law of the flat engagement in involute cylindrical gears with asymmetric profile is derived on the basis of the theorem for reversing the direction of the movement [6, 7] and allows deriving the following consequences:

Online ISSN 2256-070X
https://doi.org/10.17770/etr2021vol3.6576

Consequence I:Equal thickness of the teeth of the jointly working wheels of the gear along the poloid circle are obtained within one complete pitch angle ($2.\pi/z$) – Possibility I for generation.

This approach assumes that all variables are known in the following transcendental system:

$$\left|\begin{array}{l} inv\alpha_{tw} + inv\alpha^*_{tw} = \dfrac{2.(x_2 \pm x_1).(tg\alpha + tg\alpha^*)}{z_2 \pm z_1} + inv\alpha_t + inv\alpha^*_t \\ \dfrac{d_{tb}}{\cos\alpha_{tw}} = \dfrac{d^*_{tb}}{\cos\alpha^*_{tw}} \end{array}\right. \tag{1}$$

where:

α_{tw} and α^*_{tw} are the angles of engagement in the front section;

α_t and α^*_t – profile angles of the tool in the front section;

x_1 and x_2 – coefficient of tool displacement for the gear wheels as the internal gearing is assumed to be the equivalent wheel with external teeth.

In this correlation the „+" sign refers to external gearing while „-„ sign refers to internal gearing.

It is solved with respect to the unknown angles of engagement α_{tw} and α^*_{tw}:

$$\alpha_{tw(n+1)} = \alpha_{tw(n)} - \frac{inv\alpha_{tw(n)} + inv\left(\arccos\left(\dfrac{\cos\alpha^*_t}{\cos\alpha_t}.\cos\alpha_{tw(n)}\right)\right)}{tg^2\alpha_{tw(n)} + tg^2\left(\arccos\left(\dfrac{\cos\alpha^*_t}{\cos\alpha_t}.\cos\alpha_{tw(n)}\right)\right)} + \frac{\left\{\left[2.(x_2 \pm x_1).(tg\alpha + tg\alpha^*)/(z_2 \pm z_1)\right] + inv\alpha_t + inv\alpha^*_t\right\}}{tg^2\alpha_{tw(n)} + tg^2\left(\arccos\left(\dfrac{\cos\alpha^*_t}{\cos\alpha_t}.\cos\alpha_{tw(n)}\right)\right)} \tag{2}$$

What is important with this possibility of generation is that the sufficient condition for equality of inter axis distances is reduced to the following:

$$\frac{\cos\alpha_t}{\cos\alpha_{tw}} = \frac{\cos\alpha^*_t}{\cos\alpha^*_{tw}} \tag{3}$$

From correlation (3) it follows that this possibility of generation has the following disadvantages (fig.1):

- the profile angles of the tool α_t и α^*_t must be known in advance;
- when a large difference between the profile angles is used, the height of the tool is decreased;
- the full form of the transcendental system (1) is not used;

there is wide range of data about gears with symmetric profile, which cannot be extrapolated directly onto the asymmetric profile;

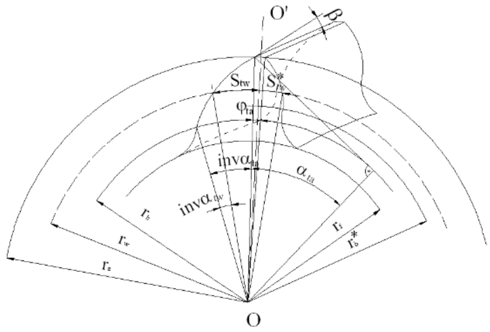

Fig. 1. General form of the tooth profile for possibility I of generation

All these disadvantages are eliminated by using a new option in the synthesis, which is determined by the following *Consequence II*: *The asymmetric gear profile can be implemented with a single tool by the method of centroid wrapping.*

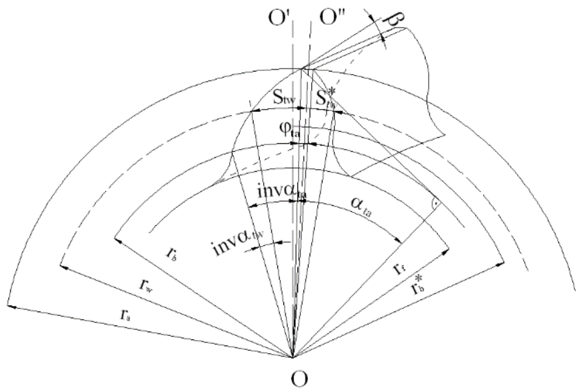

Fig. 2. Difference between the initial symmetry axis OO' and the new axis OO" for Possibility II of generation in the front section of the wheel

Based on consequence II of the basic law of engagement, two new approaches are offered for solving the transcendental system (1). For the geometrical interpretation of these two possibilities it is assumed that the full form of the transcendental system (1) is implemented, which implies that the asymmetry between the profiles is expressed by the basic circles and not only by the difference between the profile angles of the tool. This implies a difference in the "initially" accepted axis OO'. Fig. 2 shows the difference between the "initial" symmetry axis OO' and the "new" axis - OO" in the front section of the wheel. Hypothetically, we will assume that we can set the position of this new axis OO".

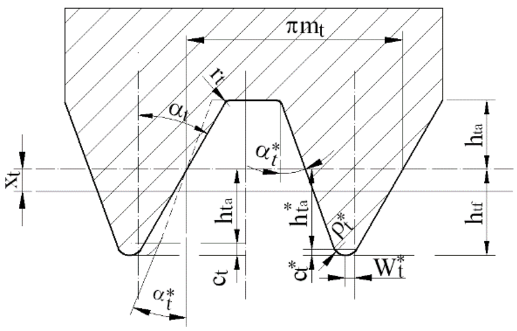

Fig. 3. Congruent output contour for possibility II of generation of asymmetric gear profile in the front section

It is assumed that the profile angles of the output contour α и α* in the normal cross-section (fig.3) are known in advance. Based on the transcendental system (1) the engagement angle α^*_{tw} is determined by the method of successive approximations [8], until the desired accuracy:

$$\alpha^*_{tw(n+1)} = \alpha^*_{tw(n)} - \frac{(z_2 \pm z_1).\left(inv\alpha_{tw} + inv\alpha^*_{tw(n)} - inv\alpha - inv\alpha^*\right)}{tg^2\alpha^*_{tw(n)}} + \frac{2.(x_2 \pm x_1).\left(tg\alpha + tg\alpha^*\right)}{tg^2\alpha^*_{tw(n)}} \tag{4}$$

The profile angle of the tool (α_t*) in the front section is determined by the sufficient condition of the basic law of engagement in asymmetric profile [9]. The general form of that other sequence of solving the system is:

$$\begin{cases} inv\alpha_{tw} = \dfrac{2.(x_2 \pm x_1).tg\alpha}{z_2 \pm z_1} + inv\alpha_t \\ \dfrac{d_{tb}}{\cos\alpha_{tw}} = \dfrac{d^*_{tb}}{\cos\alpha^*_{tw}} \\ inv\alpha^*_{tw} = \dfrac{2.(x_2 \pm x_1).\left(tg\alpha + tg\alpha^*\right)}{z_2 \pm z_1} + inv\alpha_t + inv\alpha^*_t - inv\alpha_{tw} \end{cases} \tag{5}$$

Apart from this variant of solution, the transcendental system (1) offers a third possibility for the same initial assumptions, but in a different sequence.

The engagement angle α^*_{tw} is determined based on the ratio of the main circles, which is known in advance:

$$\alpha^*_{tw} = \arccos\left(\frac{d^*_b}{d_b}\cos\alpha_{tw}\right) \tag{6}$$

The profile angle of the tool α^*_t, is determined depending on the correlation on slack-free gearing (1) as a dependent variable until the desired accuracy:

$$\alpha^*_{t(n+1)} = \alpha^*_{t(n)} - \frac{(z_2 \pm z_1).\left(inv\alpha_{tw} + inv\alpha^*_{tw} - inv\alpha_t - inv\alpha^*_{t(n)}\right)}{2.(x_2 \pm x_1).\cos^2\alpha^*_{(n)} + (z_2 \pm z_1).tg^2\alpha^*_{t(n)}} + \frac{2.(x_2 \pm x_1).\left(tg\alpha + tg\alpha^*_{(n)}\right)}{2.(x_2 \pm x_1).\cos^2\alpha^*_{(n)} + (z_2 \pm z_1).tg^2\alpha^*_{t(n)}} \tag{7}$$

The engagement angle α^*_{tw} relative to axis OO' is determined in accordance with correlation (3).

$$\begin{cases} inv\alpha_{tw} = \dfrac{2.(x_2 \pm x_1).tg\alpha}{z_2 \pm z_1} + inv\alpha_t \\ \cos\alpha^*_{tw} = \dfrac{d^*_{tb}}{d_{tb}}\cos\alpha_{tw} \\ inv\alpha^*_{tw} + inv\alpha_{tw} = \dfrac{2.(x_2 \pm x_1).\left(tg\alpha + tg\alpha^*\right)}{z_2 \pm z_1} + inv\alpha_t + inv\alpha^*_t \end{cases} \tag{8}$$

Based on this other sequence in solving transcendental correlation (3), the following Consequence III is reached: The synthesis of gears with asymmetric profile allows optimizing of the engagement of gears with symmetric profile [10].

Such optimization of the asymmetric profile based on the basic law of engagement has been implemented for output involute cylindrical gear with external engagement and the following parameters of the output contour: m=1mm, α=α*=20°, h^*_a=1, h^*_f=1,25, displacement of the contour x_1=x_2=0,5, number of teeth of wheel 1 of gear z_1=20, u=1.5, β=0 and an open profile milling method. The data about the main qualitative indicators and comparison with the output symmetric profile are given in table 1 [11].

As a basic parameter for the optimization (Table 1) the criterion of maximum front overlap ratio ε_α=max. One optimal variant has been determined without clipping at integer value of α*. This possibility of solving obtains increase of the front overlap ratio from ε_α=1,33 for symmetric profile up to ε^*_α=1,44 for asymmetric profile – possibility III.

This new optimization procedure, proposed for the first time by the authors, is graphically interpreted in the field of independent coefficients of displacement of the tool and is a two-dimensional representation of a spatial area. This is achieved through the new optimization procedure which allows a finite number of iterations to reach the required parameters of the output contour (fig.4).

This proposed new possibility III of generation has the following advantages:

- uses the available data for symmetric profile;
- allows synthesis of asymmetric profile, where the slack-free engagement is achieved relative to an axis different from the initial one;
- the limitation of the tool is eliminated, while the profile angle α* varies within the accepted proportionality between the two basic (generating) circles;

Gear	**Angles of the contour for optimized profile**	**Thickness of the teeth along the top circles**	**Overlap ratio**	**Specific slipping**
Symmetric	$\alpha^* = 20°$	$Sa_1 = 0{,}62$ $Sa_2 = 0{,}70$	$\varepsilon_\alpha = \varepsilon^*_\alpha = 1{,}33$	$\lvert\theta_a\rvert=1{,}21$ $\lvert\theta_e\rvert=1{,}17$
Asymmetric III	$\alpha^* = 15°$	$Sa_1 = 0{,}67$ $Sa_2 = 0{,}76$	$\varepsilon^*_\alpha=1{,}44$	$\lvert\theta^*_a\rvert=2{,}05$ $\lvert\theta^*_e\rvert=1{,}74$

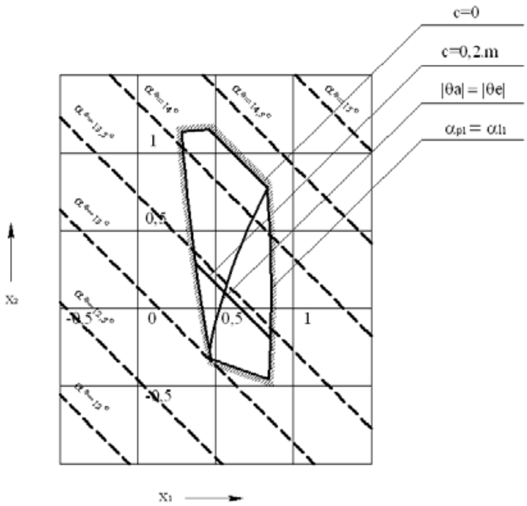

Fig. 4. Unconditional range of existence of involute cylindrical gear with asymmetry factor 1,0368 and guaranteed front overlap ratio $\varepsilon_\alpha>1{,}4$ through the whole range

From correlation (5) and (8) it follows, that possibility III for synthesis does not change the necessary and sufficient condition but complements, those by setting them in the context of the Basic law of interlocking. Therefore, the involutes generated through possibility I are equidistant to those generated through possibility III. Compared to possibility I for synthesis, possibility III offers another sequence for solving the transcendental system (1), which is a conversion of the classic theory of tooth engagement and creation of new gears with hitherto unknown qualitative and strength parameters.

III. Conclusion

Three consequences of the basic law of gear interlocking have been defined and proved, which presupposes different possibilities of generation and geometric synthesis of involute gearing with asymmetric profile of the teeth with different starting poloids of the gear wheels, as well as various improved qualitative indicators of the tooth engagement which is impossible to achieve if the teeth have symmetric profile.

The existence of three possibilities of generation of asymmetric tooth profile was proven, providing different qualitative and strength parameters at equal initial variables of the synthesis, which has been shown by example of optimization of a gear with asymmetric profile of the teeth according to the criterion of maximum front overlap ratio $\varepsilon\alpha$=max.

A new optimization procedure has been obtained and geometrically presented, allowing geometric synthesis of involute cylindrical gears with symmetric and asymmetric profiles of the teeth according to pre-set qualitative indicators through Possibility III of generation, which provides equidistant profiles at variable parameters of the tool.

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