

Brain Connections Analysis Using Graph Theory Measures

Olesja Minejeva
Riga Technical University
Riga, Latvia
o.minejeva@inbox.lv

Zigurds Markovics
Riga Technical University
Riga, Latvia
Zigurds.Markovics@rtu.lv

Nauris Zdanovskis
Riga Stradins University
Riga, Latvia
nzdhanovskis@gmail.com

Abstract—Brain is a part of the organism's complex structure that performs many functions, which are responsible for the main human abilities: to talk, to hear, to move, to see, etc. The brain consists of several areas that are not only directly connected with the different body systems, but also depend and may affect each other. Researchers and doctors are trying to summarize and visualize these relationships for an important purpose – to get the information about possible reactions of the body in case of various diseases, possibilities of recovery, risks, etc. important issues. Neurologists are looking for ways to “move” through the brain in virtual space for viewing the synapses between different areas. It might be useful to get a general idea of how brain regions are interrelated. The term “connectome”, which is the complete structural description of the brain connections, or the map of connections, is used for the common perception of brain relationships. Connectome is a network of thousands of nerve fibres that transmits signals between the special regions responsible for functions such as vision, hearing, movement and memory, and combines these functions in a system that perceives, decides and acts as a whole. So, the relationships of brain neural regions can be represented as a graph with vertices corresponding to specific areas, but edges are links between these areas. This graph can be analysed using quantitative measures, like node degree, centrality, modularity etc. This article discusses the different network measures for the connections between brain's regions. The purpose is to determine the most important areas and the role of individual connections in the general functional brain model.

Keywords— Brain network, connectome, functional connectivity, graph theory.

I. INTRODUCTION

Everyone has a unique combination of genetics, environmental impact and life experience. These factors affect the detailed “structure” of the brain, as even twins can have different levels of neural links. By arranging these connections, the researchers try to understand what could be the connectomes of different people.

Connectome is a full description of the structural connections of the brain. These connections can be between different elements of the nervous system - from neurons to whole areas of the brain [1].

Connectome is enough complex and poorly to be understood. So far, only one connectome was developed to the end. It was worm's *Caenorhabditis elegans* connectome. The problem is that this worm has only 300 neurons connected to each other by 7000 links, but the human has 100 billion times more neurons and a million times more links. [2].

There are currently several Connectivity Atlases, which, like maps, display different locations or regions of human brain. Several scientific projects are devoted to this problem. One of them is the *Brainnetome Atlas* [3]. The main goal is to explore the cerebral hierarchy by highlighting the two main elements - nodes and connections between them. The most important thing is not only to identify the structural architecture of these nodes, but to combine it with the functional consistency of the individual regions, i.e. how regions can affect each other and how the presence or interception of links can affect the functionality of the affected regions.

It turns out, that the structure of the brain can be visualized in the form of a graph. The graph structure consists of vertices or nodes and edges or connections. In different complex systems, they can display different elements and links between them, such as people and their social relationships, web sites and hyperlinks, etc. [1], [4]. In the case of the brain, regions will play the role of vertices, and the edges will implement links between these regions.

II. MATERIALS AND METHODS

Graph theory offers a variety of tools for working with complex system models that also include brain structure. So, it is possible to calculate a number of mathematical values for the graph you created and then integrate them into the complex network.

The graph type must be determined before any calculations are made. This article analyses the graph obtained from the *Brainnetome Atlas* project, where information about the brain regions and the existence of connections is provided, and shows is this region connected with this. So here it's about the so-called *undirected graph* [5]. In order to get and visualize this graph, we need data about

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the links that are gathered in one matrix, which is called *adjacency matrix*. This is a two-dimensional matrix consisting of rows and columns that reflect the names of regions (Fig. 1) [6] – [8].

| Lobe | Gyrus | Left and Right Hemisphere |
|--------------|-----------------------------|---------------------------|
| Frontal Lobe | SFG, Superior Frontal Gyrus | SFG_L(R)_7_1 |
| | | SFG_L(R)_7_2 |
| | | SFG_L(R)_7_3 |
| | | SFG_L(R)_7_4 |
| | | SFG_L(R)_7_5 |
| | | SFG_L(R)_7_6 |
| | | SFG_L(R)_7_7 |
| | MFG, Middle Frontal Gyrus | MFG_L(R)_7_1 |
| | | MFG_L(R)_7_2 |
| | | MFG_L(R)_7_3 |
| | | MFG_L(R)_7_4 |
| | | MFG_L(R)_7_5 |
| | | MFG_L(R)_7_6 |
| | | MFG_L(R)_7_7 |
| | IFG, Inferior Frontal Gyrus | IFG_L(R)_6_1 |
| | | IFG_L(R)_6_2 |
| | | IFG_L(R)_6_3 |
| | | IFG_L(R)_6_4 |

Names of brain regions from the adjacency matrix

Storage of information in the form of an adjacency matrix is usually associated with a term *density*. This parameter is equal to the ratio between the actual number of edges in the graph and the total number of possible edges [1], [5].

The names of 246 regions (SFG_L(R)_7_1, MFG_L(R)_7_7, IFG_L(R)_6_3 etc.) and two types of numbers – 0 and 1, are included in the adjacency matrix. If the edge between region *a* and region *b* exists then the corresponding matrix element is $A_{ab} = 1$, else $A_{ab} = 0$ [7]. The data is stored in .csv file format. A fragment of the adjacency matrix is shown in the Fig. 2. The number of edges connecting the vertex with others is called *node degree*. Thus, the number of “1” in the corresponding line of the adjacency matrix corresponds to the node degree of this row. Nodes with the highest degrees tend to be called *hubs* [1], [5]. On the basis of node degree we can calculate so-called *assortativity* [10], which describes the correlation between connected vertices. If this value is positive, it means that a high degree nodes prone to be connected to each other.

Another characteristic is an *average path length*. When we guess about the classic mathematical implementation, the path length is the number of edges to pass through to get from one vertex to the other. When it comes to brain topology, the path length can be used to assess the possibilities of transmitting information between the regions. Short path lengths make it quick to transfer information and reduce resource consumption during the transmitting process. This factor leads to the term „small world” [9].

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,1 SFG_L_7_1,2 SFG_R_7_1,3 SFG_L_7_2,4 SFG_R_7_2,5 SFG_L_7_3,6 SFG_R_7_3,7 SFG_L_7_4,8 SFG_R_7_4,9 SFG_L_7_5,10 SFG_R_7_5,11 SFG_L_7_6,12 SFG_R_7_6,13 SFG_L_7_7,14 SFG_R_7_7,15 MFG_L_7_1,16 MFG_R_7_1,17 MFG_L_7_2,18 MFG_R_7_2,19 MFG_L_7_3,20 MFG_R_7_3,21 MFG_L_7_4,22 MFG_R_7_4,23 MFG_L_7_5,24 MFG_R_7_5,25 MFG_L_7_6,26 MFG_R_7_6,27 MFG_L_7_7,28 MFG_R_7_7,29 IFG_L_6_1,30 IFG_R_6_1,31 IFG_L_6_2,32 IFG_R_6_2,33 IFG_L_6_3,34 IFG_R_6_3,35 IFG_L_6_4,36 IFG_R_6_4,37 IFG_L_6_5,38 IFG_R_6_5,39 IFG_L_6_6,40 IFG_R_6_6,41 IFG_L_6_7,42 IFG_R_6_7,43 IFG_L_6_8,44 IFG_R_6_8,45 IFG_L_6_9,46 IFG_R_6_9,47 IFG_L_6_10,48 IFG_R_6_10,49 IFG_L_6_11,50 IFG_R_6_11,51 IFG_L_6_12,52 IFG_R_6_12,53 IFG_L_6_13,54 IFG_R_6_13,55 IFG_L_6_14,56 IFG_R_6_14,57 IFG_L_6_15,58 IFG_R_6_15,59 IFG_L_6_16,60 IFG_R_6_16,61 IFG_L_6_17,62 IFG_R_6_17,63 IFG_L_6_18,64 IFG_R_6_18,65 IFG_L_6_19,66 IFG_R_6_19,67 IFG_L_6_20,68 IFG_R_6_20,69 IFG_L_6_21,70 IFG_R_6_21,71 IFG_L_6_22,72 IFG_R_6_22,73 IFG_L_6_23,74 IFG_R_6_23,75 IFG_L_6_24,76 IFG_R_6_24,77 IFG_L_6_25,78 IFG_R_6_25,79 IFG_L_6_26,80 IFG_R_6_26,81 IFG_L_6_27,82 IFG_R_6_27,83 IFG_L_6_28,84 IFG_R_6_28,85 IFG_L_6_29,86 IFG_R_6_29,87 IFG_L_6_30,88 IFG_R_6_30,89 IFG_L_6_31,90 IFG_R_6_31,91 IFG_L_6_32,92 IFG_R_6_32,93 IFG_L_6_33,94 IFG_R_6_33,95 IFG_L_6_34,96 IFG_R_6_34,97 IFG_L_6_35,98 IFG_R_6_35,99 IFG_L_6_36,100 IFG_R_6_36,101 IFG_L_6_37,102 IFG_R_6_37,103 IFG_L_6_38,104 IFG_R_6_38,105 IFG_L_6_39,106 IFG_R_6_39,107 IFG_L_6_40,108 IFG_R_6_40,109 IFG_L_6_41,110 IFG_R_6_41,111 IFG_L_6_42,112 IFG_R_6_42,113 IFG_L_6_43,114 IFG_R_6_43,115 IFG_L_6_44,116 IFG_R_6_44,117 IFG_L_6_45,118 IFG_R_6_45,119 IFG_L_6_46,120 IFG_R_6_46,121 IFG_L_6_47,122 IFG_R_6_47,123 IFG_L_6_48,124 IFG_R_6_48,125 IFG_L_6_49,126 IFG_R_6_49,127 IFG_L_6_50,128 IFG_R_6_50,129 IFG_L_6_51,130 IFG_R_6_51,131 IFG_L_6_52,132 IFG_R_6_52,133 IFG_L_6_53,134 IFG_R_6_53,135 IFG_L_6_54,136 IFG_R_6_54,137 IFG_L_6_55,138 IFG_R_6_55,139 IFG_L_6_56,140 IFG_R_6_56,141 IFG_L_6_57,142 IFG_R_6_57,143 IFG_L_6_58,144 IFG_R_6_58,145 IFG_L_6_59,146 IFG_R_6_59,147 IFG_L_6_60,148 IFG_R_6_60,149 IFG_L_6_61,150 IFG_R_6_61,151 IFG_L_6_62,152 IFG_R_6_62,153 IFG_L_6_63,154 IFG_R_6_63,155 IFG_L_6_64,156 IFG_R_6_64,157 IFG_L_6_65,158 IFG_R_6_65,159 IFG_L_6_66,160 IFG_R_6_66,161 IFG_L_6_67,162 IFG_R_6_67,163 IFG_L_6_68,164 IFG_R_6_68,165 IFG_L_6_69,166 IFG_R_6_69,167 IFG_L_6_70,168 IFG_R_6_70,169 IFG_L_6_71,170 IFG_R_6_71,171 IFG_L_6_72,172 IFG_R_6_72,173 IFG_L_6_73,174 IFG_R_6_73,175 IFG_L_6_74,176 IFG_R_6_74,177 IFG_L_6_75,178 IFG_R_6_75,179 IFG_L_6_76,180 IFG_R_6_76,181 IFG_L_6_77,182 IFG_R_6_77,183 IFG_L_6_78,184 IFG_R_6_78,185 IFG_L_6_79,186 IFG_R_6_79,187 IFG_L_6_80,188 IFG_R_6_80,189 IFG_L_6_81,190 IFG_R_6_81,191 IFG_L_6_82,192 IFG_R_6_82,193 IFG_L_6_83,194 IFG_R_6_83,195 IFG_L_6_84,196 IFG_R_6_84,197 IFG_L_6_85,198 IFG_R_6_85,199 IFG_L_6_86,200 IFG_R_6_86,201 IFG_L_6_87,202 IFG_R_6_87,203 IFG_L_6_88,204 IFG_R_6_88,205 IFG_L_6_89,206 IFG_R_6_89,207 IFG_L_6_90,208 IFG_R_6_90,209 IFG_L_6_91,210 IFG_R_6_91,211 IFG_L_6_92,212 IFG_R_6_92,213 IFG_L_6_93,214 IFG_R_6_93,215 IFG_L_6_94,216 IFG_R_6_94,217 IFG_L_6_95,218 IFG_R_6_95,219 IFG_L_6_96,220 IFG_R_6_96,221 IFG_L_6_97,222 IFG_R_6_97,223 IFG_L_6_98,224 IFG_R_6_98,225 IFG_L_6_99,226 IFG_R_6_99,227 IFG_L_6_100,228 IFG_R_6_100,229 IFG_L_6_101,230 IFG_R_6_101,231 IFG_L_6_102,232 IFG_R_6_102,233 IFG_L_6_103,234 IFG_R_6_103,235 IFG_L_6_104,236 IFG_R_6_104,237 IFG_L_6_105,238 IFG_R_6_105,239 IFG_L_6_106,240 IFG_R_6_106,241 IFG_L_6_107,242 IFG_R_6_107,243 IFG_L_6_108,244 IFG_R_6_108,245 IFG_L_6_109,246 IFG_R_6_109,247 IFG_L_6_110,248 IFG_R_6_110,249 IFG_L_6_111,250 IFG_R_6_111,251 IFG_L_6_112,252 IFG_R_6_112,253 IFG_L_6_113,254 IFG_R_6_113,255 IFG_L_6_114,256 IFG_R_6_114,257 IFG_L_6_115,258 IFG_R_6_115,259 IFG_L_6_116,260 IFG_R_6_116,261 IFG_L_6_117,262 IFG_R_6_117,263 IFG_L_6_118,264 IFG_R_6_118,265 IFG_L_6_119,266 IFG_R_6_119,267 IFG_L_6_120,268 IFG_R_6_120,269 IFG_L_6_121,270 IFG_R_6_121,271 IFG_L_6_122,272 IFG_R_6_122,273 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IFG_L_6_278,584 IFG_R_6_278,585 IFG_L_6_279,586 IFG_R_6_279,587 IFG_L_6_280,588 IFG_R_6_280,589 IFG_L_6_281,590 IFG_R_6_281,591 IFG_L_6_282,592 IFG_R_6_282,593 IFG_L_6_283,594 IFG_R_6_283,595 IFG_L_6_284,596 IFG_R_6_284,597 IFG_L_6_285,598 IFG_R_6_285,599 IFG_L_6_286,600 IFG_R_6_286,601 IFG_L_6_287,602 IFG_R_6_287,603 IFG_L_6_288,604 IFG_R_6_288,605 IFG_L_6_289,606 IFG_R_6_289,607 IFG_L_6_290,608 IFG_R_6_290,609 IFG_L_6_291,610 IFG_R_6_291,611 IFG_L_6_292,612 IFG_R_6_292,613 IFG_L_6_293,614 IFG_R_6_293,615 IFG_L_6_294,616 IFG_R_6_294,617 IFG_L_6_295,618 IFG_R_6_295,619 IFG_L_6_296,620 IFG_R_6_296,621 IFG_L_6_297,622 IFG_R_6_297,623 IFG_L_6_298,624 IFG_R_6_298,625 IFG_L_6_299,626 IFG_R_6_299,627 IFG_L_6_300,628 IFG_R_6_300,629 IFG_L_6_301,630 IFG_R_6_301,631 IFG_L_6_302,632 IFG_R_6_302,633 IFG_L_6_303,634 IFG_R_6_303,635 IFG_L_6_304,636 IFG_R_6_304,637 IFG_L_6_305,638 IFG_R_6_305,639 IFG_L_6_306,640 IFG_R_6_306,641 IFG_L_6_307,642 IFG_R_6_307,643 IFG_L_6_308,644 IFG_R_6_308,645 IFG_L_6_309,646 IFG_R_6_309,647 IFG_L_6_310,648 IFG_R_6_310,649 IFG_L_6_311,650 IFG_R_6_311,651 IFG_L_6_312,652 IFG_R_6_312,653 IFG_L_6_313,654 IFG_R_6_313,655 IFG_L_6_314,656 IFG_R_6_314,657 IFG_L_6_315,658 IFG_R_6_315,659 IFG_L_6_316,660 IFG_R_6_316,661 IFG_L_6_317,662 IFG_R_6_317,663 IFG_L_6_318,664 IFG_R_6_318,665 IFG_L_6_319,666 IFG_R_6_319,667 IFG_L_6_320,668 IFG_R_6_320,669 IFG_L_6_321,670 IFG_R_6_321,671 IFG_L_6_322,672 IFG_R_6_322,673 IFG_L_6_323,674 IFG_R_6_323,675 IFG_L_6_324,676 IFG_R_6_324,677 IFG_L_6_325,678 IFG_R_6_325,679 IFG_L_6_326,680 IFG_R_6_326,681 IFG_L_6_327,682 IFG_R_6_327,683 IFG_L_6_328,684 IFG_R_6_328,685 IFG_L_6_329,686 IFG_R_6_329,687 IFG_L_6_330,688 IFG_R_6_330,689 IFG_L_6_331,690 IFG_R_6_331,691 IFG_L_6_332,692 IFG_R_6_332,693 IFG_L_6_333,694 IFG_R_6_333,695 IFG_L_6_334,696 IFG_R_6_334,697 IFG_L_6_335,698 IFG_R_6_335,699 IFG_L_6_336,700 IFG_R_6_336,701 IFG_L_6_337,702 IFG_R_6_337,703 IFG_L_6_338,704 IFG_R_6_338,705 IFG_L_6_339,706 IFG_R_6_339,707 IFG_L_6_340,708 IFG_R_6_340,709 IFG_L_6_341,710 IFG_R_6_341,711 IFG_L_6_342,712 IFG_R_6_342,713 IFG_L_6_343,714 IFG_R_6_343,715 IFG_L_6_344,716 IFG_R_6_344,717 IFG_L_6_345,718 IFG_R_6_345,719 IFG_L_6_346,720 IFG_R_6_346,721 IFG_L_6_347,722 IFG_R_6_347,723 IFG_L_6_348,724 IFG_R_6_348,725 IFG_L_6_349,726 IFG_R_6_349,727 IFG_L_6_350,728 IFG_R_6_350,729 IFG_L_6_351,730 IFG_R_6_351,731 IFG_L_6_352,732 IFG_R_6_352,733 IFG_L_6_353,734 IFG_R_6_353,735 IFG_L_6_354,736 IFG_R_6_354,737 IFG_L_6_355,738 IFG_R_6_355,739 IFG_L_6_356,740 IFG_R_6_356,741 IFG_L_6_357,742 IFG_R_6_357,743 IFG_L_6_358,744 IFG_R_6_358,745 IFG_L_6_359,746 IFG_R_6_359,747 IFG_L_6_360,748 IFG_R_6_360,749 IFG_L_6_361,750 IFG_R_6_361,751 IFG_L_6_362,752 IFG_R_6_362,753 IFG_L_6_363,754 IFG_R_6_363,755 IFG_L_6_364,756 IFG_R_6_364,757 IFG_L_6_365,758 IFG_R_6_365,759 IFG_L_6_366,760 IFG_R_6_366,761 IFG_L_6_367,762 IFG_R_6_367,763 IFG_L_6_368,764 IFG_R_6_368,765 IFG_L_6_369,766 IFG_R_6_369,767 IFG_L_6_370,768 IFG_R_6_370,769 IFG_L_6_371,770 IFG_R_6_371,771 IFG_L_6_372,772 IFG_R_6_372,773 IFG_L_6_373,774 IFG_R_6_373,775 IFG_L_6_374,776 IFG_R_6_374,777 IFG_L_6_375,778 IFG_R_6_375,779 IFG_L_6_376,780 IFG_R_6_376,781 IFG_L_6_377,782 IFG_R_6_377,783 IFG_L_6_378,784 IFG_R_6_378,785 IFG_L_6_379,786 IFG_R_6_379,787 IFG_L_6_380,788 IFG_R_6_380,789 IFG_L_6_381,790 IFG_R_6_381,791 IFG_L_6_382,792 IFG_R_6_382,793 IFG_L_6_383,794 IFG_R_6_383,795 IFG_L_6_384,796 IFG_R_6_384,797 IFG_L_6_385,798 IFG_R_6_385,799 IFG_L_6_386,800 IFG_R_6_386,801 IFG_L_6_387,802 IFG_R_6_387,803 IFG_L_6_388,804 IFG_R_6_388,805 IFG_L_6_389,806 IFG_R_6_389,807 IFG_L_6_390,808 IFG_R_6_390,809 IFG_L_6_391,810 IFG_R_6_391,811 IFG_L_6_392,812 IFG_R_6_392,813 IFG_L_6_393,814 IFG_R_6_393,815 IFG_L_6_394,816 IFG_R_6_394,817 IFG_L_6_395,818 IFG_R_6_395,819 IFG_L_6_396,820 IFG_R_6_396,821 IFG_L_6_397,822 IFG_R_6_397,823 IFG_L_6_398,824 IFG_R_6_398,825 IFG_L_6_399,826 IFG_R_6_399,827 IFG_L_6_400,828 IFG_R_6_400,829 IFG_L_6_401,830 IFG_R_6_401,831 IFG_L_6_402,832 IFG_R_6_402,833 IFG_L_6_403,834 IFG_R_6_403,835 IFG_L_6_404,836 IFG_R_6_404,837 IFG_L_6_405,838 IFG_R_6_405,839 IFG_L_6_406,840 IFG_R_6_406,841 IFG_L_6_407,842 IFG_R_6_407,843 IFG_L_6_408,844 IFG_R_6_408,845 IFG_L_6_409,846 IFG_R_6_409,847 IFG_L_6_410,848 IFG_R_6_410,849 IFG_L_6_411,850 IFG_R_6_411,851 IFG_L_6_412,852 IFG_R_6_412,853 IFG_L_6_413,854 IFG_R_6_413,855 IFG_L_6_414,856 IFG_R_6_414,857 IFG_L_6_415,858 IFG_R_6_415,859 IFG_L_6_416,860 IFG_R_6_416,861 IFG_L_6_417,862 IFG_R_6_417,863 IFG_L_6_418,864 IFG_R_6_418,865 IFG_L_6_419,866 IFG_R_6_419,867 IFG_L_6_420,868 IFG_R_6_420,869 IFG_L_6_421,870 IFG_R_6_421,871 IFG_L_6_422,872 IFG_R_6_422,873 IFG_L_6_423,874 IFG_R_6_423,875 IFG_L_6_424,876 IFG_R_6_424,877 IFG_L_6_425,878 IFG_R_6_425,879 IFG_L_6_426,880 IFG_R_6_426,881 IFG_L_6_427,882 IFG_R_6_427,883 IFG_L_6_428,884 IFG_R_6_428,885 IFG_L_6_429,886 IFG_R_6_4
```

spread across the all network.

Of course, the described graph analysis parameters are only a small part of graph theory tools. But with regard to brain connections, they give a general idea of the interrelations and interactions between the regions. There is a wide range of software that can be used for mathematical and statistical processing of this graph and for calculating parameters of this graph. It can be divided into two groups: universal mathematical packages (MATHCAD, MATLAB, Wolfram Mathematica etc.), containing tool groups for working with graphs, and specialized computer programs (for example., Gephi), designed for purposeful processing and analysis of graphs.

This paper describes *NetworkX* software package. It is free software for creating and analysis of complex networks. The program for graph processing is created in *Python* programming language. Launching the created program, it represents parameters described above that gives the information about the links between brain regions.

III. RESULTS AND DISCUSSION

NetworkX program allows you to import data from .csv file format. Such file is an adjacency matrix obtained from *Brainnetome Atlas*, where information about connections between 246 brain regions is stored. Program code for importing file:

```
#to import matrix from csv file
from numpy import genfromtxt
import numpy as np
mydata = genfromtxt("C:/Python/matrix.csv",
delimiter=',')
print(mydata)
```

Created program *mydata.py* is launches in the *cmd* window. Data is displayed on the screen (Fig. 3). As the matrix is large, only its part is visible on the screen. The names of the regions were imported as words *nan*.

```
[nan nan nan ... nan nan nan]
[nan 0. 1. ... 1. 1. 1.]
[nan 1. 0. ... 1. 1. 1.]
...
[nan 1. 1. ... 0. 1. 1.]
[nan 1. 1. ... 1. 0. 1.]
[nan 1. 1. ... 1. 1. 0.]
```

Imported matrix from .csv file0

In order to “pull” the adjacency matrix from the imported data, some lines are added to the code:

```
#adjacency matrix from imported data
adjacency = mydata[1:,1:]
print(adjacency)
```

The adjacency matrix is output to the screen (Fig. 4).

```
[[0. 1. 1. ... 1. 1. 1.]
 [1. 0. 1. ... 1. 1. 1.]
 [1. 1. 0. ... 0. 1. 1.]
 ...
 [1. 1. 0. ... 0. 1. 1.]
 [1. 1. 1. ... 1. 0. 1.]
 [1. 1. 1. ... 1. 1. 0.]
```

Fig. 2. Adjacency matrix

NetworkX programme is a console application, but there are additional extension packages that also allow visualization. Different types of graphical objects can be constructed in *NetworkX* by connecting the library *Matplotlib*. Connecting libraries, now we can create a graph from the adjacency matrix and display it:

```
#graph drawing
import networkx as nx
import matplotlib.pyplot as plt
G=nx.from_numpy_matrix(adjacency)
nx.draw(G)
plt.show()
```

When we run this code, a new pop-up window with a graph’s image opens (Fig. 5). Since the number of vertexes and edges in the graph is large enough and the graph is wide, because the brain connections form a complex system, then the resulting drawing is complicated and non-informative. The vertexes and edges merge, so the graph cannot be used for further analysing.

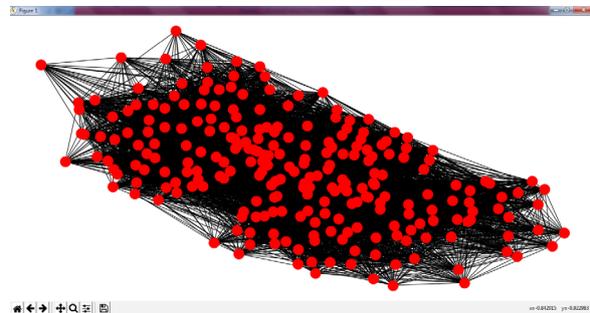


Fig. 3. Graph of brain connections

In turn, *NetworkX* has libraries with different graph parameters that can be calculated.

The first simplest parameter is node degree. It has a built-in function for calculating it - *Graph.degree*. Here are two options. If we analyse a particular vertex or some vertexes, which are similar to brain regions, then we can add a vertex number as a parameter to this function, for example:

```
G.degree[10] # node's 10 degree
list(G.degree([0, 1, 2]))# degrees of 3 nodes
```

Alternatively, we can output a whole array of vertexes with the appropriate degrees to the screen (Fig. 6).

```
[<0, 115>, <1, 107>, <2, 46>, <3, 38>, <4, 91>, <5, 83>, <6, 44>, <7, 96>, <8, 110>, <9, 113>, <10, 113>, <11, 113>, <12, 113>, <13, 106>, <14, 49>, <15, 42>, <16, 66>, <17, 60>, <18, 60>, <19, 60>, <20, 60>, <21, 64>, <22, 49>, <23, 49>, <24, 104>, <25, 114>, <26, 42>, <27, 42>, <28, 69>, <29, 33>, <30, 70>, <31, 34>, <32, 77>, <33, 76>, <34, 84>, <35, 84>, <36, 88>, <37, 88>, <38, 41>, <39, 88>, <40, 79>, <41, 76>, <42, 53>, <43, 45>, <44, 126>, <45, 49>, <46, 107>, <47, 50>, <48, 90>, <49, 93>, <50, 63>, <51, 56>, <52, 59>, <53, 47>, <54, 58>, <55, 70>, <56, 72>, <57, 60>, <58, 55>, <59, 64>, <60, 105>, <61, 65>, <62, 98>, <63, 66>, <64, 83>, <65, 67>, <66, 80>, <67, 68>, <68, 114>, <69, 72>, <70, 84>, <71, 77>, <72, 74>, <73, 54>, <74, 62>, <75, 110>, <76, 43>, <77, 80>, <78, 55>, <79, 81>, <80, 68>, <81, 74>, <82, 74>, <83, 77>, <84, 56>, <85, 62>, <86, 88>, <87, 60>, <88, 73>, <89, 75>, <90, 54>, <91, 54>, <92, 76>, <93, 55>, <94, 96>, <95, 81>, <96, 44>, <97, 56>, <98, 56>, <99, 53>, <100, 76>, <101, 103>, <102, 112>, <103, 104>, <104, 82>, <105, 74>, <106, 80>, <107, 73>, <108, 72>, <109, 111>, <110, 71>, <111, 67>, <112, 73>, <113, 68>, <114, 78>, <115, 118>, <116, 96>, <117, 100>, <118, 32>, <119, 34>, <120, 76>, <121, 49>, <122, 83>, <123, 126>, <124, 87>, <125, 94>, <126, 21>, <127, 129>, <128, 49>, <129, 133>, <130, 53>, <131, 134>, <132, 66>, <133, 61>, <134, 62>, <135, 40>, <136, 140>, <137, 63>, <138, 141>, <139, 56>, <140, 54>, <141, 38>, <142, 153>, <143, 136>, <144, 146>, <145, 149>, <146, 134>, <147, 150>, <148, 138>, <149, 138>, <150, 138>, <151, 138>, <152, 61>, <153, 64>, <154, 53>, <155, 57>
```

Fig. 4. Degrees of graph nodes

Other parameters were also calculated for graph analysis:

```
#Assortativity
r=nx.degree_assortativity_coefficient(G)
print("%3.1f"%r)
#Clustering coefficient
print(nx.average_clustering(G))
#Average path length in the graph
print(nx.average_shortest_path_length(G))
#Density
print(nx.density(G))
#Centrality
print(nx.degree_centrality(G))
print(nx.betweenness_centrality(G))
print(nx.closeness_centrality(G))
```

Launching such a program a number of numerical values is obtained and summarized in Table I.

TABLE I. ESTIMATED GRAPH PARAMETERS

| Graph Parameters | Value |
|-------------------------|-------|
| Assortativity | 0 |
| Clustering coefficients | 0.69 |
| Average path length | 1.65 |
| Density | 0.37 |
| Diameter | 3 |

The first parameter – assortativity – equal to 0. This coefficient indicates whether high-level vertexes tend to interact with the same or similar vertexes. Since the calculated coefficient is equal to 0, this means that this graph can be called as *non-assortative graph*. In such graphs there is no correlation between vertexes that give the name of such complex systems - *uncorrelated networks*.

If you pay attention to the fragmentation of the graph, let's look at the value of the clustering coefficient obtained. Numerically it is equal to 0.69. It is difficult to assess whether this figure is considered to be low or too high. From a theoretical point of view „small – world” graphs have high clustering. The coefficients is between 0 and 1. This value reaches 0.69 for the given graph, that is noticeably larger than half, and means, that brain regions form separate groups or subgroups, in which almost all vertexes have connections to others. A more detailed view can be given by cluster analysis methods.

The characteristics of the information exchange in the graph should be analyzed separately. The obtained value of density is 0.37. After this relationship, especially in the context of brain activity, it is impossible to say objectively and unambiguously whether the transmission of information between the regions is good or bad. Of course, this ratio is not approaching 1.0, where there are all the possible edges between nodes, but on the other hand it has gone far enough from 0. To create a full scene, let's look at how you can get from one node to the other. Average path length in the graph is 1.65. So you can transfer impulses from region *a* to region *b*, offending

only a few regions on the road. Maximum path length directly from one node to other – diameter – is equal to 3. Therefore it also confirms the relatively efficient and fast flow of information in the graph.

IV. CONCLUSIONS

A method to describe brain connectivity using graph theory measurements is described in this article. There are a lot of parameters that can be calculated for the graph of connections between the brain regions. First of all, we can conclude that brain activity is a very complicated system and can be represented as a wide graph with a big number of vertexes and edges. So, it is difficult to analyse whole graph.

Brain connections graph can be called “small - world” system. The “small - world” graphs tend to contain subgraphs that have connections between almost all the nodes in them. This follows from the definition of such a property of the graph as a high coefficient of clustering. Secondly, most pairs of vertexes are connected at least in one short way. Thus, some of brain regions tend to make separate groups. It means that signals through connections will affect only one group of regions but not whole brain.

Mathematical analysis has been performed using *NetworkX* software. Calculated values have created a common presentation about the analysable graph. But for more accurate analysis, more mathematic software packages can be used to compare obtained results.

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