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On Mathematical Modelling of the 2-D Filtration Problem in Porous axial symmetrical cylinder

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Abstract. In this paper we study diffusion and convection filtration problem of one substance through the pores of a porous material which may absorb and immobilize some of the diffusing substances. As an example we consider round cylinder with filtration process in the axial direction. The cylinder is filled with sorbent i.e. absorbent material that passed through dirty water or liquid solutions. We can derive the system of two partial differential equations (PDEs). One equation is expressing the rate of change of concentration of water in the pores of the sorbent and the other - the rate of change of concentration in the sorbent or kinetically equation for absorption. The approximation of corresponding initial boundary value problem of the system of PDEs is based on the conservative averaging method (CAM). This procedure allows reducing the 2-D axis-symmetrical mass transfer problem described by a system of PDEs to initial value problem for a system of ordinary differential equations (ODEs) of the first order.

Keywords: absorption, analytical and numerical solution. diffusion problem, filtration, sorbents, special splines.

I. INTRODUCTION

The task of sufficient accuracy numerical simulation of quickly solution 3-D problems for mathematical physics in multilayered media is important in known areas of the applied sciences – heat transfer in multilayered media, for example, calculation of the concentration of metals in peat layers [16], the heat and moisture transfer processes in the porous multilayered media layer, for example, mathematical modelling of moistening and drying process in the wood-block [1].

For this purpose we consider two methods: special finite difference schemes and conservative averaging method (CAM) by using integral parabolic and exponential splines.

Therefore, the CAM is considered in the present article; too, the finite-difference method is used for solving the 1-D initial value problem for system of ODEs due to obtain the solution of the 3-D initial value problem.

A. Buikis was developed different assumptions for CAM along the vertical coordinate in the Cartesian coordinates using parabolic splines [3], [11]. We are expanding the usage of splines method with integral parabolic and exponential splines [16], [2] in addition, not only in Cartesian coordinates, but also in cylindrical coordinates too [1], [10], if it requires the model under consideration.

The study of hydrodynamic flow and heat transfer through a porous media becomes much more interesting due to its vast applications [8], [2] and [10]. Many mathematical models are developed for the analysis of such processes, for example, mathematical models of moisture movement in wood, when the wood is considered as porous media [1], [10].

II. MATHERIALS AND METHODS

1. A mathematical model

Filtration is the separation process of removing solid particles, microorganisms or droplets from a liquid or a gas by depositing them on a filter medium [15]. This paper deals with filtration processes of solid-liquid mixtures (suspensions, slurries, sledges) [12]. For adsorption kinetics we use linear Henry [9] and nonlinear Langmuir [3], [13], [14] sorption isotherms.

In [3] a contaminant transport model with Langmuir sorption under non-equilibrium conditions which is described by two coupled equations – advective-dispersion equation and non-equilibrium sorption equations is considered. In this paper we study the filtration process with diffusion and convection in the domain

$$\Omega = \{(r,z,\phi): 0 \le r \le R, 0 \le z \le L, 0 \le \phi \le 2\pi\}.$$

This domain Ω consists of porous material, where through the pores of filter moves incompressible

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liquid - pollutants in z-direction. This problem has practical meaning and also theoretical interest in mathematical physics problems in which several small parameters appear. These parameters are connected with some geometrical dimensions in the problem and also with the relations between the coefficients of the equations. We will consider the nonstationary axis-symmetrical problem of the linear filtration theory.

We can derive two equations; one is the adsorbed phase of concentration $a(r,z,t)$ for the pollutants which are absorbed per unit volume and per unit time. The other equation is the aqueous phase of pollutants concentration $u(r,z,t)$ in sorbent pores. Then convection and diffusion PDEs in the cylindrical coordinates are in the following form [17], [5]:

$$\begin{cases} \left(\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(D_r r\dfrac{\partial u}{\partial r}\right)+D_z\dfrac{\partial^2 u}{\partial z^2}\right)+V_0\dfrac{\partial Cu}{\partial z}=m\dfrac{\partial u}{\partial t}+\dfrac{\partial a}{\partial t}, \\ \dfrac{\partial u}{\partial t}=\beta(u-\tilde{u}),\ r\in[0,R], z\in[0,L], t>0, \end{cases}$$

where $a=\tilde{u}/\gamma$ is the expression for linear isotherm of Henry, D_r, D_z are the transversal un tangential diffusion coefficients or the dispersion coefficients, $V_0=const$ is the pore water velocity in z-direction, m is the fraction of the total volume of the material occupied by pores, $\tilde{u}$ is concentration of pollutants, which is in local equilibrium conditions $\partial a/\partial t=0$ with the amount of liquid sorbet, t is the time, β is the kinetically coefficient or the sorption rate constant, $1/\gamma$ is the Henry coefficient for the sorbent characteristic.

We assume that all coefficients in the PDEs are assumed constant and independent of concentration. For nonlinear sorption we have $a=\tilde{u}/(\lambda(1+p\tilde{u}))$, which is referred to as Langmuir isotherm, where p is positive parameter (for $p=0$ we have Henry isotherm). For the initial condition for $t=0$ we give $u(r,z,0)=0$, $a(r,z,0)=0$. We use following boundary conditions:

$$\begin{cases} \dfrac{\partial u(0,z,t)}{\partial r}=\dfrac{\partial u(0,z,t)}{\partial r}=0,\ u(R,z,t)=a(R,z,t)=0, \\ u(r,L,t)=u_0(t)=U_0(1-\tanh(\alpha t)),\ \dfrac{\partial u(r,0,t)}{\partial z}=0, \\ \dfrac{\partial a(r,0,t)}{\partial z}=0, \end{cases} \quad (1)$$

where $\alpha=const$, $U_0=const$. The concentration u on the inlet is depending on t.

For predetermined parameters $u_1=u/U_0$, $a_1=a\gamma/U_0$, $\tilde{u}_1=\tilde{u}/U_0$, $t_1=t\gamma\beta$ we have following system (3):

$$\begin{cases} \left(\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(D_r r\dfrac{\partial u_1}{\partial r}\right)+D_z\dfrac{\partial^2 u_1}{\partial z^2}\right)+V_0\dfrac{\partial u_1}{\partial z}=m\gamma\beta\dfrac{\partial u_1}{\partial t_1}+\beta\dfrac{\partial a_1}{\partial t_1}, \\ \dfrac{\partial a_1}{\partial t_1}=u_1-f(a_1),\ r\in[0,R], z\in[0,L], t>0, \end{cases}$$

where $f(a_1)=a_1/(1-\tilde{p}a_1)$, $\tilde{p}=pU_0$, $u_1(r,L,t_1)=u_{0z}(t_1)=1-\tanh(\alpha_1 t_1)$, $\alpha_1=\alpha/\gamma\beta$.

For $\tilde{p}=0$ we have a linear Henry isotherm.

2. The conservative averaged method in z-direction

We consider conservative averaging method (CAM) of the special integral splines with hyperbolic trigonometrically functions for solving the initial-boundary-value problem in z-direction [6]. This procedure allows reducing the 2-D problem in r,z-directions to a 1D problem in r direction. Using CAM in z-direction with parameter a_z we have

$$u_1(r,z,t_1)=u_v(r,t_1)+m_z(r,t_1)\frac{0.5L\sinh(a_z(z-0.5L))}{\sinh(0.5a_zL)}+$$

$$e_z(r,t_1)\left(\frac{\cosh(a_z(z-0.5L))-A_z}{8\sinh^2(a_zL/4)}\right),$$

where, $u_v(r,t_1)=L^{-1}\int_0^L u_1(r,z,t_1)dz$,

$$A_z=\frac{\sinh(a_zL/2)}{a_zL/2}.$$

The parameter a_z can be choosing for minimizing the maximal error. If the parameter $a_z>0$ tends to zero then the limit is the integral parabolic spline (A. Buikis [4]), because of $A_z\to 1$:

$$u_1(r,z,t_1)=u_v+m_z(z-0.5L)+e_z\left(\frac{(z-0.5L)^2}{L^2}-\frac{1}{12}\right)$$

The unknown functions $m_z=m_z(r,t_1), e_z=e_z(r,t_1)$ can be determined from conditions:

1) for $z=0$ $m_zd_z-e_zk=0$, $m_z=e_zp_1$, $p_1=k/d$, $u_1(r,0,t_1)=u_v-m_zL/2+e_zb,$

2) for $z=L$, $u_{0z}=u_v+m_zL/2+e_zb$, $e_z=(u_{0z}-u_v)/g0$, where $d=0.5La_z\coth(0.5a_zL)$, $k=0.25a_z\coth(0.25a_zL)$, $b=(\cosh(a_zL/2)-A_z)/(8\sinh^2(a_zL/4))$, $g0=b+0.5L\ p_1$.

Now the 1-D initial-value problem (3) is in the following form

$$\begin{cases} m\gamma\beta\dfrac{\partial u_v}{\partial t_1}+\beta\dfrac{\partial a_v}{\partial t_1}=\dfrac{1}{r}\left(\dfrac{\partial}{\partial r}\left(D_r r\dfrac{\partial u_v}{\partial r}\right)\right)+ \\ a0^2(u_{0z}-u_v), \\ \dfrac{\partial a_v}{\partial t_1}=u_v-f(a_v),\ r\in[0,R],L],\ t>0, \\ \dfrac{\partial u_v(0,t_1)}{\partial r}=0,\ \dfrac{\partial a_v(0,t_1)}{\partial r}=0,\ u_v(R,t_1)= \\ a_v(R,t_1)=0,\quad u_v(r,0)=a_v(r,0)=0, \end{cases} \quad (4)$$

Where $a_v(r,t_1)=L^{-1}\int_0^L a_1(r,z,t_1)dz$,

$a0^2=\left(2D_z\dfrac{k}{L}+V_0p_1\right)/g0$, $f(a_v)=a_v/(1-\tilde{p}a_v)$- here we assume that the averaging of the nonlinear term $f(a_v)$ does not change its form.

3. The conservative averaged method in r-direction

Using averaged method in r-direction with parameters a_r we have

$u_v(r,t_1)=u_{vv}(t_1)+m_r(t_1)f_m(r)+e_r(t_1)f_e(r)$,

where

$$f_m(r)=\frac{0.25R^2(a_r)^2\sinh(a_r(r-0.5R))}{\sinh(0.5a_rR)(d_1-1)}-1,$$

$$f_e(r)=\frac{\cosh(a_r(r-0.5R))-A_r}{8\sinh^2(a_rR/4)},$$

$$\frac{2}{R^2}\int_0^R rf_m(r)dr=\frac{2}{R^2}\int_0^R rf_e(r)dr=0,$$

$$A_r=\frac{\sinh(a_rR/2)}{a_rR/2},\ d_1=0.5Ra_r\coth(0.5a_rR).$$

We can use following values of parameters

$a_r=a0\sqrt{1/D_r}$.

If the parameter $a_r>0$ tends to zero then the limit is the integral parabolic spline:

$$u_v(r,t_1)=u_{vv}+m_r\left(\frac{6}{R}(r-0.5R)-1\right)+ e_r\left(\frac{(r-0.5R)^2}{R^2}-\frac{1}{12}\right).$$

From boundary conditions (4) follows the unknown coefficients-functions:

1) for $r=0$ $m_rdr-e_rk_r=0$ or $m_r=e_rp_r$

2) for $r=R$, $0=u_{vv}+m_rb_m+e_rb_e$ or

$e_r=-u_{vv}/g_r$, where $d_r=\dfrac{0.5d_1R(a_r)^2}{d_1-1}$,

$k_r=0.25a_r\coth(0.25a_rR)$, $p_r=k_r/d_r$,

$b_e=(\cosh(a_rR/2)-A_r)/(8\sinh^2(a_rR/4))$,

$b_m=\dfrac{0.25R^2(a_r)^2}{d_1-1}-1$, $g_r=b_e+p_rb_m$.

Now the 1-D initial- value problem (4) is in the form of following ODEs system:

$$\begin{cases} m\gamma\beta\dfrac{\partial u_{vv}}{\partial t_1}+\beta\dfrac{\partial a_{vv}}{\partial t_1}=-b0^2u_{vv}(t_1)+ \\ a0^2(u_{0z}(t_1)-u_{vv}(t_1)), \\ \dfrac{\partial a_{vv}}{\partial t_1}=u_{vv}(t_1)-f(a_{vv}),\ t_1>0, \\ u_{vv}(0)=a_{vv}(0)=0, \end{cases} \quad (5)$$

where $f(a_{vv})=a_{vv}(t_1)/(1-\tilde{p}a_{vv}(t_1))$,

$a_{vv}(t_1)=\dfrac{2}{R^2}\int_0^R ra_v(r,t_1)dr$, $b0^2=\dfrac{4D_rk_r}{Rg_r}$.

We rewrite the1-D initial-value problem for system ODEs (5) in following normal form:

$$\begin{cases} \dot{u}_{vv}(t_1)=b_{11}u_{vv}(t_1)+b_{12}f(a_{vv})+f_1u_{0z}(t_1), \\ \dot{a}_{vv}(t_1)=b_{21}u_{vv}(t_1)+b_{22}f(a_{vv}),\ t_1>0, \\ u_{vv}(0)=0,\ a_{vv}(0)=0, \end{cases} \quad (6)$$

where $b_{11}=-\dfrac{\beta+b0^2+a0^2}{m\gamma\beta}$, $b_{12}=\dfrac{1}{m\gamma}$, $b_{21}=1$,

$b_{22}=-1$, $f_1=-\dfrac{a0^2}{m\gamma\beta}$.

If $\tilde{p}=0$ then we have the following vector form of linear ODEs system:

$\dot{W}(t_1)=AW(t_1)+F,\ W(0)=0$, (7)

where $W(t_1),F(t_1)$ are the 2-order vector-column with elements $(u_{vv}(t_1),a_{vv}(t_1))$, $(f_1u_{0z}(t_1),0)$.

A is the 2-order matrix $A=\begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22}\end{pmatrix}$.

The averaged linear and nonlinear ($\tilde{p}\neq 0$) solution we can obtain with Matlab solver " ode15s".

4. Backward orientation for CAM

For estimation the parameters a_z,a_r we use also backward orientation for CAM – first of all, we do CAM in r-direction and then in the in z-direction.

Then in r-direction we have

$u_1(r,z,t_1)=u_v(z,t_1)+m_r(z,t_1)f_m(r)+$
$e_r(z,t_1)f_e(r)$.

From boundary conditions we have $e_r=-u_v/g_r$, $m_r=e_rp_r$ and the problem (3) is in following form (8):

$$\begin{cases} m\gamma\beta\dfrac{\partial u_v}{\partial t_1}+\beta\dfrac{\partial a_v}{\partial t_1}=D_z\dfrac{\partial^2 u_v}{\partial z^2}+V_0\dfrac{\partial u_v}{\partial z}-b0^2u_v, \\ \dfrac{\partial a_v}{\partial t_1}=u_v-f(a_v),\, z\in[0,L], z\in[0,L], t>0, \\ \dfrac{\partial u_v(0,t_1)}{\partial z}=0, \dfrac{\partial a_v(0,t_1)}{\partial z}=0, u_v(L,t_1)=u_{0z} \\ a_v(L,t_1)=0, u_v(z,0)=a_v(z,0)=0 \end{cases}$$

where

$$a_v(z,t_1)=\frac{2}{R^2}\int_0^R ra_1(r,z,t_1)dr,\; b0^2=\frac{4D_rk_r}{Rg_r},$$

$f(a_v)=a_v/(1-\tilde{p}a_v)$.

Using CAM in z-direction we have

$$a_v(z,t_1)=u_{vv}(t_1)+m_z(t_1)\frac{0.5L\sinh(a_z(z-0.5L))}{\sinh(0.5a_zL)}+$$

$$e_z(t_1)\left(\frac{\cosh(a_z(z-0.5L))-A_z}{8\sinh^2(a_zL/4)}\right),$$

where

$$u_{vv}(t_1)=L^{-1}\int_0^L u_v(z,t_1)dz,\; a_z=b0\sqrt{1/D_z}.$$

From boundary conditions we have

$e_z=(u_{0z}-u_{vv})/g0$, $m_z=e_zp_1$ and the problem (8) is in the form of (5), where

$$a_{vv}(t_1)=L^{-1}\int_0^L a_v(z,t_1)dz,$$

$$a0^2=\left(2D_z\frac{k}{L}+V_0p_1\right)/g0.$$

Therefore we have in every CAM orentation obtained two algebraic equations for determine the spline parameters in following form

$$a_r=f_1(a_r)=a0\sqrt{1/D_r}$$

$$a_z=f_2(a_z)=b0\sqrt{1/D_z}.$$

The optimal parameters we can obtained by solution these equations with method of simple iteration. For $V_0\neq 0$ it is possible usage of exponential type spline for equations (8) in method CAM [7].

5. CAM for model equations

For approbation CAM in r-direction and estimated the parameter a_r we consider model stationary 1-D boundary-value problem in following form:

$$\begin{cases} Dr^{-1}(ru'(r))'-a0^2u(r)=F_0,\, r\in[0,R], \\ u'(0)=0, u(R)=u_0, \end{cases} \tag{9}$$

where $u_0, F_0, a0>0, D>0$ are given constants. The analytical solution is $u(r)=C_1I_0(a_1r)-f_1$, $C_1=\dfrac{u_0+f_1}{I_0(a_1R)}$, where $I_0'(0)=I_1(0)=0$, I_0', I_1 are the modified Bessel functions, $f_1=F_0/a0^2$, $a_1=a0/\sqrt{D}$.

Using averaged method in r-direction with parameter a_r we have $u(r)=u_v+mf_m(r)+ef_e(r)$, where

$$m=\frac{(u_0-u_v)p_r}{g_r},\; e=\frac{u_0-u_v}{g_r},$$

$$u_v=\frac{4Du_0k_r-F_0g_rR}{4Dk_r+a0^2g_rR}.$$

For $D_1=1, F_0=-10,\; a0=2, a_r=2, u_0=1,\; R=5$ we have following maximal error:

1) 1.278 for parabolic spline ($a_r=10^{-4}$),
2) 0.0021 for hyperbolic spline.

III. RESULTS AND DISCUSSION

6. Some numerical results

Experimental data have been obtained studying the filtration process through hemp shives using the adsorption column "Adsorption CE 583" [12] at the Chemistry, biology and biotechnology research centre laboratory of Faculty of Engineering of RTA.

The results of calculations are obtained by MATLAB. We use the discrete grid value

$$t_n=n\frac{t_f}{N_t},\; n=\overline{0,N_t},\; z_i=i\frac{L}{N_z}, i=\overline{0,N_z},$$

$$r_j=j\frac{R}{N_r},$$

$j=\overline{0,N_r}$, $N_z=10, N_t=50, N_r=30, t_f=5;50$, $R=0.15[m]$, $L=1[m]$, and parameters $U_0=25\left[\frac{g}{l}\right]$, $\beta=1;3$, $\gamma=1;2$, $m=0.4$, $D_r=10^{-4}\left[\frac{m^2}{\min}\right]$, $D_z=5\cdot10^{-4}\left[\frac{m^2}{\min}\right]$, $V_0=0.1\left[\frac{m}{\min}\right]$, $\alpha_1=0.2$, $\tilde{p}=0;0.1;1;5;10, t_1\in[0,t_f]$. For $\beta=3$, $\gamma=1$, $a_z=10.00$, $t_f=50$, $\tilde{p}=0$ (dimensional final time is $\dfrac{t_f}{\beta\gamma}=50/3[\min]$) we obtain with direct CAM orientation $a_r=34.155$ (the results of calculations are represented in Figs.1-4 with backward CAM orientation $a_z=11.53$, with direct CAM orientation $a_r=34.0312$, with backward CAM orientation $a_z=11.5176$ (Figs. 5, 6) and with direct CAM orientation $a_r=34.0312$ (we have quickly convergent iteration process).

The maximal values of $u(r,z,t_f)=0.0198$, $u_{vv}(t)=0.0706$, $a_{vv}(t)=0.0698$ and $u_{vv}(t_f)=0.013$ are equal for both CAM orientation.

The maximal calculated dimensional value of liquid concentration in the final time ($0.50[g/l]$) is good (acceptable for practical problems) comparing with experimentally obtained concentration ($0.54[g/l]$).

The matrix A has following eigenvalues : A $\lambda_1=-3.61$, $\lambda_2=-0.042$. It was obtained, that in outlet of the domain $0.5L \le z \le L$ the concentration u is small and:

1) The averaging concentration of u_v for $r=0$ is decrising in the time with maximal value by $t_1=5$ (Fig. 1),

2) The concentration u for $t_1=50$ is maximal by $r=0$ and is incrising in z-direction (Fig. 2, Fig. 3),

3) The averaging concentrations of u_{vv} and a_{vv} are equal for both CAM orientation and different depending on the time, $a_{vv}>u_{vv}$ only for $t_1>10$,

4) The averaging concentration for CAM in r-direction of $u_v(z,t_f)$ is maximal by $z=0$ and is decreasing in z-direction (Fig. 6).

The maximal values of $u_v(r,t_f)$, $u_v(0,t)$, $u(r,z,t_f)$, $u_{vv}(t)$, $a_{vv}(t)$ and $u_{vv}(t_f)$ for different $\tilde{p}$ are represented in Table 1.

Table 1: The maximal values of $u_v(r,t_f)$, $u_v(0,t)$, $u(r,z,t_f)$, $u_{vv}(t)$, $a_{vv}(t)$ and $u_{vv}(t_f)$ depending on $\tilde{p}$

$\tilde{p}$	$u_v(r,t_f)$	$u_v(0,t)$	$u(r,z,t_f)$	$u_{vv}(t)$	$a_{vv}(t)$	$u_{vv}(t_f)$
0	.0180	.1025	.0200	.0711	.0702	.013
0.1	.0179	.1029	.0199	.0714	.0700	.013
1.0	.0172	.1067	.0191	.0740	.0683	.012
5.0	.0144	.1249	.0160	.0865	.0603	.010
10	.0118	.1474	.0131	.1022	.0505	.008
1.0^*	.0214	.0863	.0237	.0598	.0561	.015
1.0_*	.0012	.2400	.0013	.1604	.1405	.0008

We can see, that with increasing $\tilde{p}$ the filtration process is faster (see $u_v(r,t_f)$, $u_{vv}(t)$, $a_{vv}(t)$, $u_{vv}(t_f)$), but the maximum of concentration is increasing (see $u_v(0,t), u_{vv}(t)$). In the present table by $\tilde{p}=1.0^*$ there are maximal values for $\beta=3, \gamma=2$ ($\lambda_1=-2.29$, $\lambda_2=-0.033$).

We can see that the filtration process is slow.

For $\tilde{p}=1.0_*$ and $\beta=1, \gamma=1$ ($\lambda_1=-3.84$, $\lambda_2=-0.119$) the filtration process is faster.

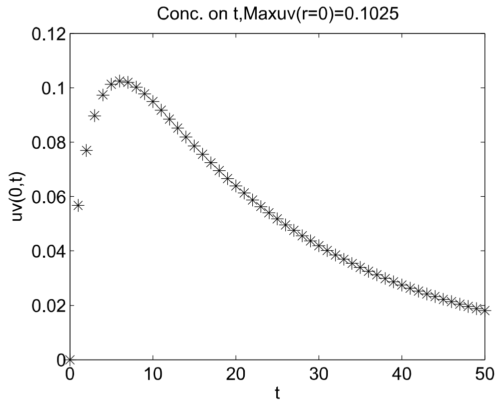

Fig. 1. Averaging concentration u_v depending on t_1 for $r=0$

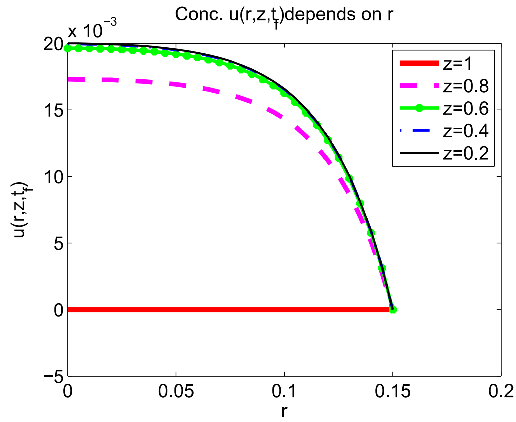

Fig. 2. Concentration $u(r,z,t_f)$ profile depending on z for $t_f=50$

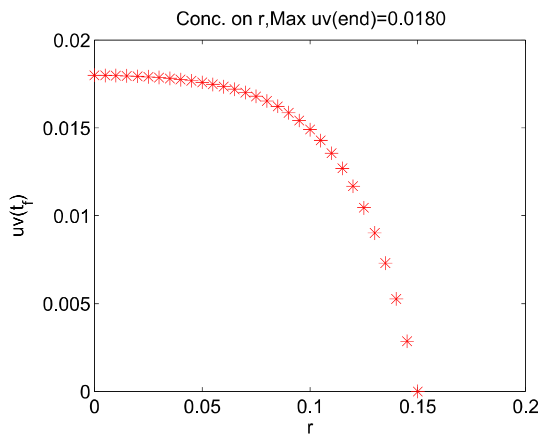

Fig. 3. Averaging concentration $u_v(r,t_f)$ depending on r for $t_f=50$

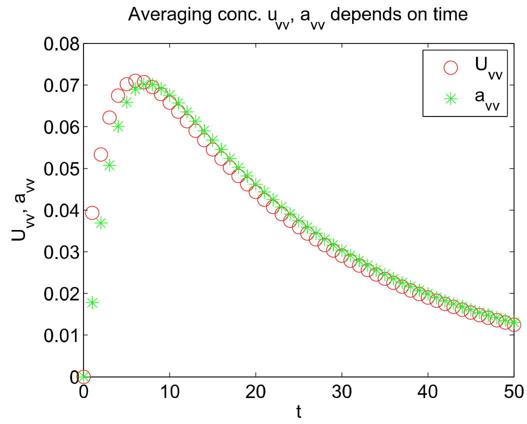

Fig. 4. Averaging concentration $u_{vv}(t_1)$ and $a_{vv}(t_1)$ depending on t_1

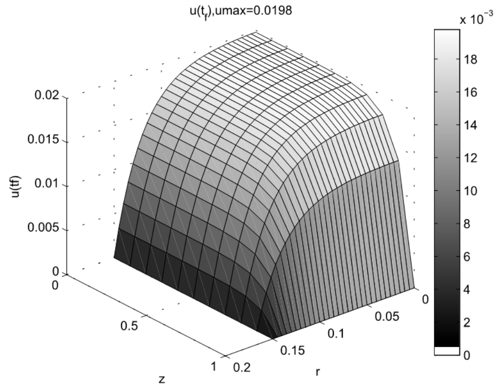

Fig. 5. Concentration $u(r,z,t_f)$ for $t_f = 50$

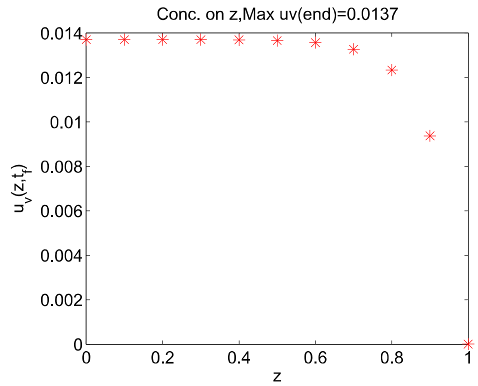

Fig. 6. Averaging concentration $u_v(z,t_f)$ depending on z for $t_f = 50$

IV. CONCLUSIONS

The approximation of corresponding initial boundary value problem of the system of PDEs is based on the conservative averaging method (CAM), where the new hyperbolic type splines are used.

For these splines the best parameter for minimal error is calculated using the direct and backward orientation for CAM. Numerical experiments confirmed the correctness of the best parameter calculation using a convergent iteration process.

The problem of the system of 3D PDEs with constant coefficients is approximated on the initial value problem of a system of ODEs of the first order. The 1-D differential and discrete problems are solved analytically.

The maximal calculated dimensional value of liquid concentration in the final time was compared with experimentally obtained concentration. It was observed in the results of good agreement that is acceptable practice.

Such a mathematical model allows us to obtain analytical solution with a simple engineering algorithm for mass transfer equations for modelling the filtration process.

The mathematical model can be used under consideration filtration process modelling - to determine the impurity concentration in the solution of filtration depending on the time.

More generally, it allows you to calculate the saturation of the filtering material, depending on the time.

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