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Mathematical Model of Friction Coefficient Determination for Lubricated Surfaces

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Abstract. **This article reviews mathematical model for the determination of friction coefficient for sliding friction pair for lubricated surfaces in case of friction of boundary lubrication. The given case examines contact of an absolutely smooth ball and flat rough surface taking into consideration properties of the material, surface roughness parameters, as well as kinematic viscosity and density of lubricating material. The model refers to widely spread ball-on-disc type tribometer measurements for ball and plane contact.**

Keywords: **friction coefficient, boundary friction, sliding friction.**

I INTRODUCTION

This article examines a mathematical model for the determination of friction coefficient for sliding friction pairs of lubricated surfaces. In the particular model contact of absolutely smooth ball and flat rough surface is regarded as a friction pair taking into consideration mechanical properties of the material and surface roughness parameters, as well as kinematic viscosity and density of lubricating material. The model refers to widely spread ball-on-disc type tribometer measurements where ball is in the contact with plane. Specification of tribometer meets tribological measurement standard [1]. In the given case mathematical modelling of friction process is being carried out, which helps to describe theoretically the effect of additive to parameter influencing friction coefficient, the mathematical model is based on the experimental work described in the publication [2].

II MATHEMATICAL MODEL OF FRICTION COEFFICIENT DETERMINATION FOR LUBRICATED SURFACES

Rotary moving parts, lubricated surface friction is shown by Strybeck curve [3] the character of which is given on Fig. 1. It shows a simplified version of the change of friction coefficient V_d depending on the *Hersy* Index $\frac{V_d \cdot n}{q}$, where V_d – dynamic viscosity of oil, n – speed of rotation, q – load per an area unit. The Strybeck curve is divided into three parts and each part characterises a separate friction mode:

I – hydrodynamic friction,
II – mixed friction,
III –friction of boundary lubrication.

In mode I two hard surfaces are separated by an uninterrupted oil layer, where thickness of oil layer h is more than the height of surface roughness R_t.

Since in this case there is no direct contact between surfaces 1 and 3 it, can be regarded that there are no friction processes between surfaces. Part II of the curve that characterises mixed friction shows that only a part of load is received by oil layer and partly also by peaks of roughness of both surfaces. Thickness of oil film h is approximately similar to the height of surface roughness R_t.

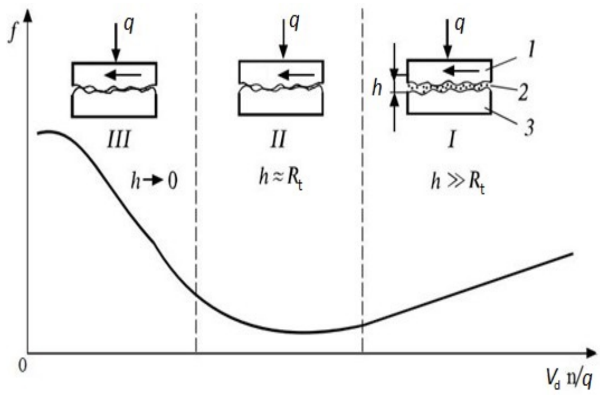

Fig. 1. Diagram of Strybeck curve and zones of lubrication modes (h - thickness of lubrication layer, R_t – height of roughness).

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In case of part III of the curve, i.e. in case of boundary lubrication thickness of lubricating material being in contact turns very thin and load is fully received by surface roughness peaks that are covered with a thin oil layer.

Based on the parameters characterising the Strybeck curve we can write the respective friction coefficient formula:

$$f = \alpha \cdot \frac{V_d \cdot n}{q} \quad (1.1)$$

where V_d - dynamic viscosity;
n – rotation speed;
q – load per an area unit;
α - coefficient of correction determined experimentally;

In practice when determining viscosity of lubricating material at different temperatures the most often there is being used kinematic viscosity V_k, which is determined using the following formula:

$$V_k = \frac{V_d}{\rho} \quad (1.2)$$

where ρ – density of lubricating material;
V_k – kinematic viscosity;

Thus the friction coefficient can be rewritten by the following formula.

$$f = \alpha \cdot \frac{V_k \cdot \rho \cdot n}{q} \quad (1.3)$$

Formula (1.3) shows that friction coefficient depends on kinematic viscosity V_k, density of lubricating material, rotation speed n and load q. Studying the effect of oil additive the only variable value is kinematic viscosity V_k because other parameters are characteristic quantities of experiment.

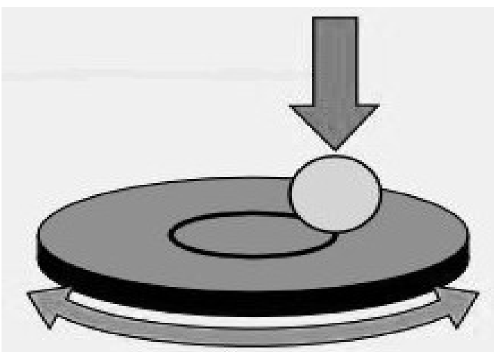

Fig.2. Contact diagram of friction surfaces according to tribometer.

The given paper envisages studying of the effect of oil additives by tribometer *CSM Instruments*, the design of which envisages contact of ball with a flat surface, therefore it is necessary to study this contact (see Fig. 2). Formula (1.3) comprises parameter q, which can be determined applying the contact provisions of friction surfaces.

III CONTACT OF ABSOLUTELY SMOOTH BALL AND FLAT ROUGH SURFACE

The surface contact theory envisages an area arising when solid body deforms rough surface [3];

$$\eta = \frac{A_r}{A_a}\ , \quad (1.4)$$

where A_r – actual contact area including roughness;
A_a –nominal area to be determined according to component dimensions.

With flat components the nominal area is equal to nominal dimensions of contacting surfaces. The nominal area of ball in contact with a plane will be the area of side surface of ball settling that is in contact with material (Fig. 3)

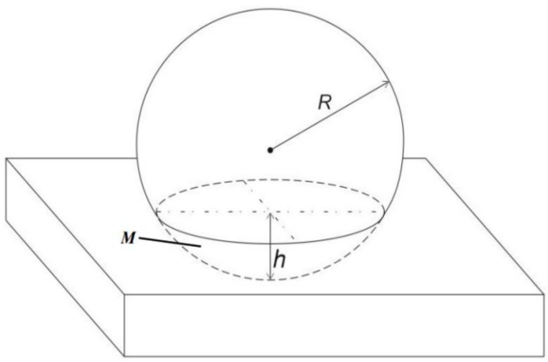

Fig.3 . Contact diagram of settled ball

According to [4] the side area of the ball M is equal to

$$M = 2\pi Rh \quad (1.5)$$

where R – ball radius;
h – height of the settled part;

In *CSM* tribometer a ball having a constant radius (R=3mm) has contact with a rough flat surface. In the existing contact theory such characteristic dimensions of contact have not been established. Therefore we will replace ball - plane contact with the contact of two planes. For this pupose we use graphical simulation of contacts (Fig.4)

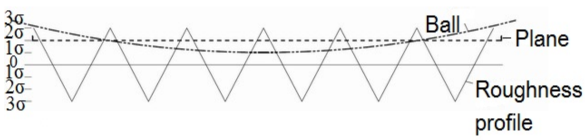

Fig. 4 Contact of ball with rough flat surface

This simultion compares the contact of ball with a rough flat surfac if the top of the ball reaches the level of surface roughness 1σ counted from the mid-line with the contact of a plane and rough surface, but plane during contact reaches level 2σ counting from the mid-line.

The result shows that at the roughness parameter R_a = 0,12-0,18 μm, contact area of ball – rough plane differs from an ideal plane - rough plane contact not more than by 5%.

It should be noted that for the purpose of visuality Fig.4 gives a schematic picture of rough surface, yet simulation of actual contact includes actul character of roughness with correct roughness step parameters.

Since the surface roughness Ra of the experimental sample of the given paper satisfies this provision we can use formulas obtained in this work [5] in case of contact of two plate.

In case of an elastic contact, arising when tribometer's steel ball contacts with a flat disk load per an area unit q can be calculated according to formula;

$$q_{el} = k_q^{el} \frac{Sa}{R_{Sm1} \cdot \theta} F_1(\gamma), \qquad (1.6)$$

where Sa – 3D surface roughness mean arithmetic deviation ;

R_{Sm1} - surface roughness steps perpendicular to the direction of surface treatment (in general case);

θ – elastic contact constant.

$$\theta = \frac{1-\mu^2}{\pi E} \qquad (1.7)$$

where $\mu -$ Poisson's ratio of deformed material.

E – elasticity modulus of deformed material.,

k_q^{el} - coefficient considering surface roughness anisotropy;

$$k_q^{el} = \frac{2}{3} \sqrt[4]{\frac{\pi}{2}} \cdot \frac{\pi}{K(e)} \left[\frac{E(e)}{K(e)}\right]^{\frac{1}{2}} \cdot \frac{(1-e^2)^{-\frac{1}{8}}}{[1+(1-e^2)^{\frac{3}{4}}]^{\frac{1}{2}}} \qquad (1.8)$$

where e – eccentricity of contact area;

K(e) and E(e) – elliptic integrals [4];

$$e^2 = 1 - c^{\frac{8}{3}} \;, \qquad (1.9)$$

Value c is obtained from the relation of surface roughness steps;

$$c = \frac{R_{Sm1}}{R_{Sm2}} \qquad (1.10)$$

R_{Sm1}, R_{Sm2} – roughness steps perpendicular to and along treatment directions.

$F_1(\gamma)$ - tabulated function of parameter γ where;

γ – deformation level counted from the mid - plane of roughness.

Since the load envisaged in the experiment on the sample is 3N (see 1.3), it can be predicted, that in the flat sample deformations, take place in the upper layers, i.e. deformation level $\gamma \geq 2$.

At $\gamma \geq 2$, according to [6]

$F_1(\gamma) \leq \frac{1}{40\gamma} \leq \frac{1}{40 \cdot 2} \leq \frac{1}{80}$;

In the given case formula (1.6) is considerably simplified, because surfaces have isotropic roughness, where c=1 and e=0 , and value of coefficient k_q^{el} *is* k_q^{el} =0,85 .

Inserting the above values in formula (1.7)

$$q_{el} = 0{,}01 \frac{Sa}{R_{Sm} \cdot \theta} \qquad (1.11)$$

Inserting formula (1.11) in the expression (1.4) we obtain the final formula for the calculation of friction coefficient:

$$f = 1{,}7 \cdot \alpha \frac{V_k \cdot \rho \cdot n \cdot R_{Sm} \cdot \theta}{S_a} \qquad (1.12)$$

Values included in the formula (1.12) have the following measuring units:

V_k- kinematic viscosity (m^2/s),
ρ – density (kg/m^3),
n – rotation speed (1/s),
R_{Sm} – surface roughness step parameter (μm),
S_a - surface roughness (μm),
θ – elastic contact constant (Pa^{-1});

At the given measuring units we get non-dimensional value of friction coefficient f.

To compare the experiment with the theory we use formula (1.12). This formula includes non-dimensional coefficient a characterising sample friction in case when oil has no additives.

According to formula (1.12) we get the following expression:

$$\alpha = \frac{f \cdot S_a}{1{,}7 \cdot V_k \cdot \rho \cdot n \cdot R_{Sm} \cdot \theta} \qquad (1.13)$$

IV CONCLUSIONS

The given mathematical model, can be used, for the comparison of results obtained during friction coefficient experiments with theoretical values. Taking into consideration the parameter of kinematic viscosity of lubricating material, at different concentrations of additives, determining viscosity experimentally. It can be concluded, from the model that influence of additives, can be strongly affected by the kinematic viscosity of lubricating material, which can affect the friction coefficient of lubricated friction pair.

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