INSTABILITY MODELING OF FINANCIAL PYRAMIDS
FINANŠU PIRAMĪDU NESTABILITĀTES MODELĒŠANA

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Abstract. *The financial structures that make use of money flow for "easy money" or cheating purpose are called financial pyramids. Recently financial pyramids intensively penetrates IT area. It is rather suitable way of the fraud. Money flow modeling and activity analysis of such financial systems allows identifying financial pyramids and taking necessary means of precautions. In the other hand even investing companies that function normally when market conditions changes (e.g. interest rates) eventually might become financial pyramid. Modeling of financial pyramids allows identifying signs of such instability.*

Keywords: *financial investments, financial pyramids, limited resources, money flow modelling.*

Introduction

Financial structures that use money flows for cheating purpose usually are called financial pyramids or financial bubble. As well snow bowl, chain letters, games, matrix, multilevel trade systems usually are called financial pyramids (FP). Recently financial pyramids intensively penetrates IT area. We can see many different FP in the internet websites. Money flow modeling and research of these systems, allows identifying signs of such instability.

There are several approaches towards mathematical description of FP in Literature.

Not giving a definition of a bubble the authors [1] mentioned that financial bubbles are "movements in the price, apparently unjustified by information available at the time, taking the form of a rapid grow followed by a burst or at least a sharp decline". In the paper the effectiveness of the market is assumed and a quite strong assumption is made: all market participants have one and the same information after "announcement" of prices.

Others authors [2] the game approach is used. A financial bubble is being modeled as stochastic incomplete information game between the Ponzi firm and population of individuals. It is supposed that the Ponzi firm knows all its movies and movies of the population, but individuals know only their own movies and of several their acquaintances.

S.V.Dubovsky [3, 4, 5] introduced a model of a financial bubble, in which different cases of the growth of the outstanding total face-value of the bubble securities in circulation or total value of current sales are given as scenarios, which are monotone growing functions of time.

The goal of our research is to analyze FP models, to introduce the mathematical models for its recognition. This problem is very distantly researched. **The object** of the work is existing FP, its methods and models. The paper is prepared using comparable analysis, mathematical analysis of scientific literature and summing-up methods.

The authors describe FP of investment companies. Such companies often organize only the capturing of money and promise high interest rates. At first the company pays these interest rates, and at the end the loss of money follows.

The simplified model of financial pyramid

Money flows are complicated, and to generalize them into united model is rather complicate. First we analyze a simple case of FP money flow [6]. Lets assume, that the Organizer of FP collets money. He promises 20% of interest rate per month (792 percentages interest rate per year). An investment can not be repossessed for 5 month. At first clients of FP put equal sums of money a. Lets find when will be collected the biggest sum of money and when it will not be enough of this sum to pay the percentages. S_n presents the Sum of money, which Organizer will collect after n months. Let's do mathematical model of such activity and build a pyramid.

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Further are presented S_n values after one, two, three and etc. months, till money theoretical be enough for the percentage payment. The example presents for how long it will be enough of accumulated money for percentage payment:

$S_1 = a$

$S_2 = a + 0{,}8a = 1{,}8a$

$S_3 = a + 0{,}8a + 0{,}6a = 2{,}4a$

$S_4 = a + 0{,}8a + 0{,}6a + 0{,}4a = 2{,}8a$

$S_5 = a + 0{,}8a + 0{,}6a + 0{,}4a + 0{,}2a = 3a$

$S_6 = a + 0{,}8a + 0{,}6a + 0{,}4a + 0{,}2a + 0 = 3a$

$S_7 = a + 0{,}8a + 0{,}6a + 0{,}4a + 0{,}2a + 0 - 0{,}2a = 2{,}8a$

$S_8 = a + 0{,}8a + 0{,}6a + 0{,}4a + 0{,}2a + 0 - 0{,}2a - 0{,}4a = 2{,}4a$

$S_9 = a + 0{,}8a + 0{,}6a + 0{,}4a + 0{,}2a + 0 - 0{,}2a - 0{,}4a - 0{,}6a = 1{,}8a$

$S_{10} = a + 0{,}8a + 0{,}6a + 0{,}4a + 0{,}2a + 0 - 0{,}2a - 0{,}4a - 0{,}6a - 0{,}8a = 1{,}0a$

$S_{11} = a + 0{,}8a + 0{,}6a + 0{,}4a + 0{,}2a + 0 - 0{,}2a - 0{,}4a - 0{,}6a - 0{,}8a - a = 0$

An example shows that paying 20% per month the Organizer will run out of money per 11 month. This will not happen as all the money will be spent for paying interest. The maximum sum he can reach on the 5th and 6th periods. But later on the sum is declining and becomes equal to 0.

Accumulative money value dependence from time and returned percentage rate.

In this example we are investigating the members of money flow, which have a fixed size. Now lets as each month β percentages (calculated by hundredth) from initial contribution are paid to depositors. Then in n month cumulative sum will be:

$S_n = a + (1-\beta)a + (1-2\beta)a + ... + (1-(n-1)\beta)a$.

In the right of equality is arithmetic progression. Lets find the sum of firsts n members of progression and get S_n value.

$$S_n = \frac{a\,n}{2}(2 + \beta(1-n)) \qquad (1)$$

Here S_n is a Cumulative Sum of money which the Organizer gets after n periods (months), a – sum of money contributed in each period (month) start, β – active percentage (hundredth) calculated from initial (principal) Sum.

With reference to formula (1) we found the cumulated capital dependence on accumulation time, (in month), when β rate of percentages paid to clients are different.

This is obvious that when the paid percentage rate is growing then time of accumulation period is declining.

Theoretically FP can exist till it has accumulated money (until S_n >0). After solving the equation $S_n = \frac{a\,n}{2}(2 + \beta(1-n)) > 0$ (when n >0), we get that FP could exist for the following time period: $n \le \frac{2}{\beta} + 1$. If we would assume that β=0.2 we would get that n<=11. That lets us to confirm our calculations and results of the figure1.

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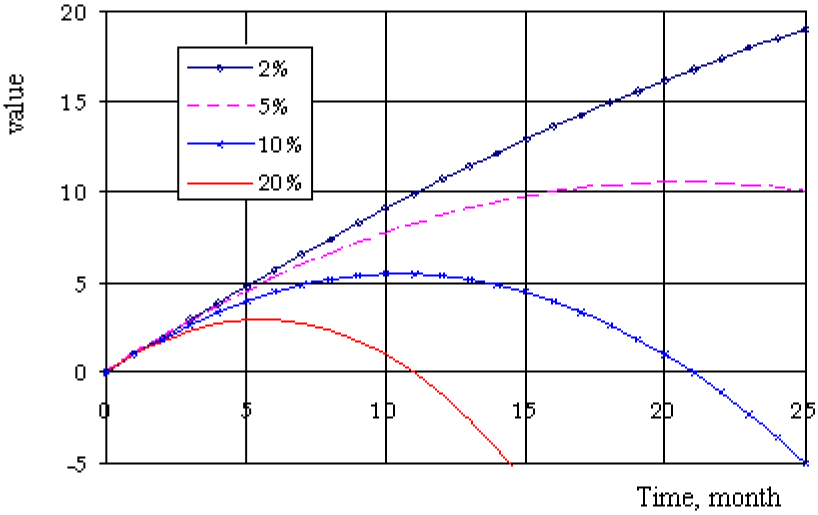

Fig. 1. **The dependence of FP accumulated capital from the time, with different interest rates**

In the calculations accumulation of money was a discrete process. More natural and more adequate process which represents real situations would be steady accumulation. **Let variable** ***n*** **let's be continuous**. We note that formula (1) is equation of parabola. After making a transformation of this equation we got $S_n = \frac{a(2+\beta)}{2}n - \frac{a\beta}{2}n^2$.

The branches of the parabola are go down. The maximum of parabola coincide with its summit. Based on known formula of parabolas summit abscissa calculation we find the biggest value of function (1) which is equal to the variable *n*.

$$n = \frac{1}{\beta} + \frac{1}{2}$$

It this is the time when FP has accumulated the largest amount of money. For example, when β=0.2 the largest sum of money will be accumulated after 5,5 month.

The same result we would get while analyzing the functions (1) extremums using fluxion.

FP with variable money flow members

Now let's analyze the case when contributed money sums are not fixed [7]. After the firsts largess interest payments the amount of contributions begins to change: to grow or decline.

The grow accumulation. Let's discuss the case of grow accumulation. We can say that number of contributions grow in geometric progression. At first lets assume that q>1. Then on the first month the contributed sum will be a, that is $S_1 = a$,

On the second month - $S_2 = a + a(1-\beta)q$

On the third $S_3 = a + a(1-\beta)q + a(1-2\beta)q^2$

At the end on month *n* the accumulated value is

$$S_n = a + a(1-\beta)q + a(1-2\beta)q^2 + ... + a(1-(n-1)\beta)q^{n-1}$$

That could be written as $S_n = a\frac{q^n - 1}{q - 1} - a \cdot \beta \sum_{i=1}^{n-1} i \cdot q^i$ (2)

If q=1 (sums of contribution are fixed sizes) we would get formula (1) $S_n = a\left(n - \beta\sum_{i=1}^{n-1} i\right)$

After analyzing the equation (2) (like on equation (1)), one more time we can see the instability of FP.

The results of equation (2) are presented in the figure 2. There is shown the case when interest of 20% from principal value is paid for each period (month), and one contributed sums are fixed size (q=1). When contribution is growing then maximum sum is accumulated in the same time

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period. The value of sum grows too. However lifetime of FP is going down, because accumulated sums are parcel out to defray rather high interest rates.

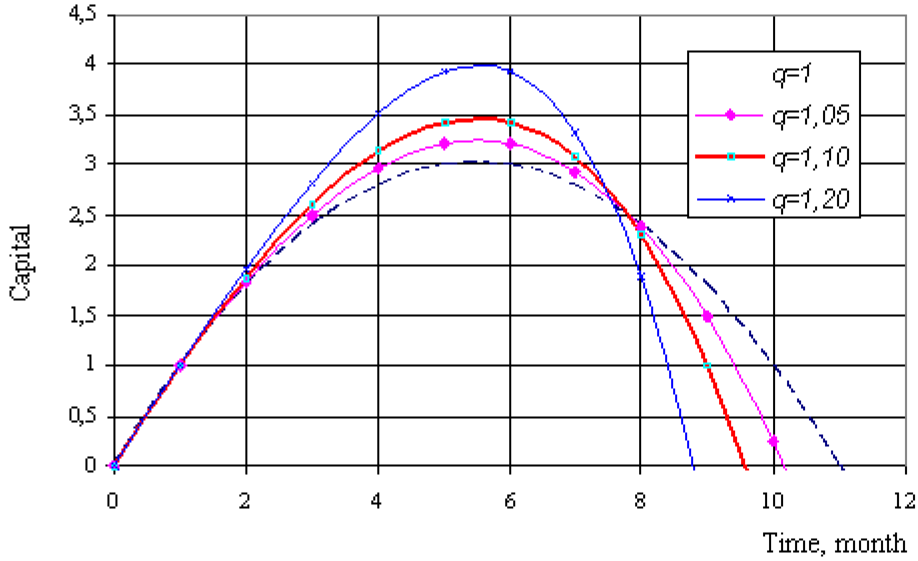

Fig. 2. **The dependence of FP accumulated capital from the time, when interest rate 20% (the grow of accumulation)**

FP lifetime has two phases (stages):
1. in the first stage accumulated capital is growing;
2. in the second one – declining.

If q=1 then the grow phase and the decline phase are symmetric: terms of both phases are equal. But if q>1 then the second phase becomes shorter than the first one.

Here we have got sudden conclusion: the bigger is the accumulated sum, the faster is it's overspend. FP which grows more quick, exists shorter than others, which principal grow is more slow. When the denominator of geometric progression grow, then grows the skew of phases: the second phase decline and conversely.

The declining accumulation. Based on the analysis of accumulation cases, we can make an assumption that decrease of denominator of progression lets exist FP for longer time. Lets denominator of progression be q<1. In the figure 3 is illustrated the dependence of accumulated capital to the cumulated time, when the denominator of progression increase from 1 to 0.7. Here the same as in earlier cases basic value of interest rate is 20 percentages (β = 20 %) per month. All contributions are fixed sizes (q=1). We saw that FP which has basic parameters exits for the shortest time. When denominator of progression decline we saw the second phase of FP lifetime sudden becomes longer. That means that such FP can exists very long time. Its lifetime in the instance could be equal to investment companies lifetime period.

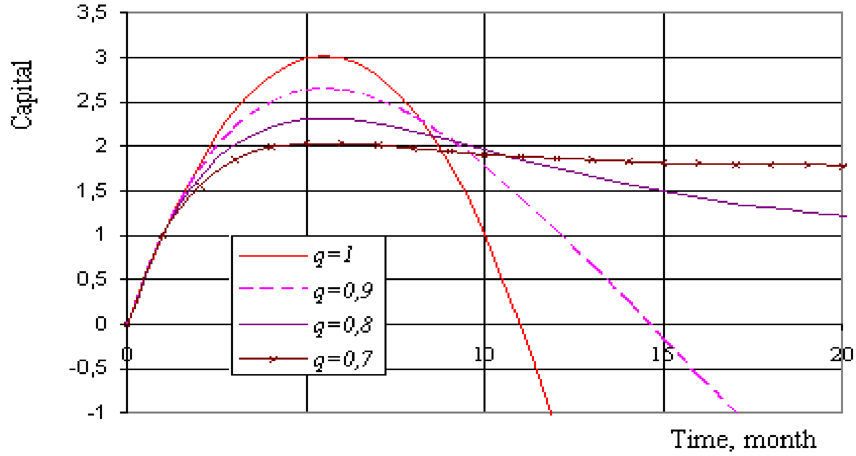

Fig. 3. **The dependence of FP accumulated capital from the time, when interest rate 20% (the decline of accumulation)**

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In all these cases we analyzed money flows and not value of time. Because FP simply doesn't make any investment. Now let's implicate the time factor.
Lets FP work like investing company which is **investing the capital with interest rate of month *i***. Here $r = 1+i$ is coefficient of capital grow. Then (accumulated value) of **future value** of FP we can describe like:

$$S_n = a\cdot r^n + a(1-\beta)q\cdot r^{n-1} + a(1-2\beta)q^2\cdot r^{n-2} + ... + a(1-(n-1)\beta)q^{n-1}\cdot r + a(1-n\beta)q^n \qquad (3)$$

Here S_n – future value of accumulated sum of money, n- cumulated number of periods, a –sum of money contributed at each period beginning, β – active percentage calculated from principal Sum (in this case could be dividend rate), q- coefficient of contribute size variation (grow, decline), r- coefficient of invest contribution grow size with interest rate i ($r = 1 + i$).
In this expression determinant value has proportion of coefficients q and r. It is not hard to see that if $q=r$ then equation (3) becomes equation (1) with further multiplier (factor) r^n. Then accumulated sum we can write like:

$$S_n = a\sum_{m=0}^{n}(1-m\beta)\cdot q^m r^{n-m}\,. \qquad (4)$$

Here m is serial number of money flow member.
We analyze accumulating model (5) and find dependences of accumulated value to cumulated time, when other FP parameters are different. Lets take fixed cumulated time equal to 50 periods (n=50), the interest rate 10 percentages and fixed amount of contribute equal to one (a = 1).

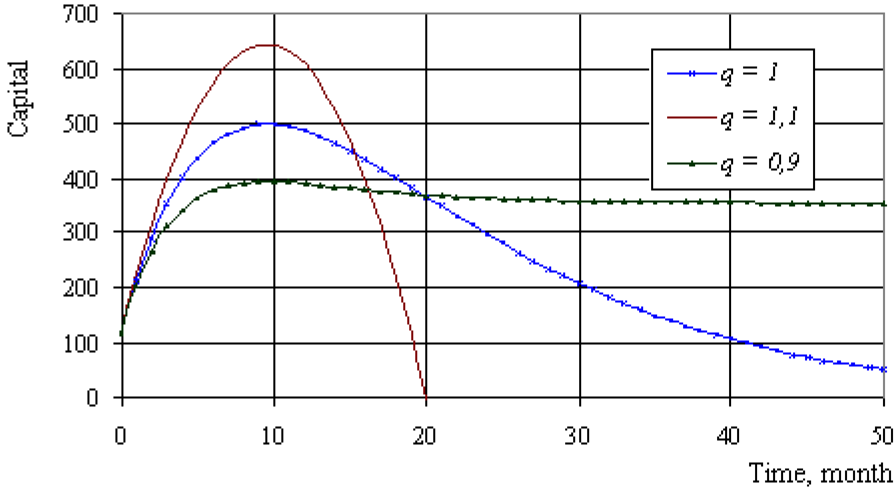

Fig. 4. **The dependence of future value of accumulated capital from the time, when *a=1; r=1.1; n=50; β=10%***

In the figure 4 we can see the dependence of future value of accumulated capital from time of the cumulate when number of contributions is different. We note the shortest lifetime of FP is at the period when number of contributions (q) is largest and is equal to coefficient of grow of amount of contribution r ($q = r = 1.1$). In the case when number of contributions is fixed (q=1), the amount of accumulated capital is slowly declining, because β value is rather high (β =0.1). In the third case FP becomes stable, when the number of contributions constantly declines (q =0.9). We can maintain that FP becomes (usual, normal) investing company. Such company has quite stable money.
There are image dependences of future value of accumulated capital from different number of contributions. It is presented in the figure 5. In this case it is taken higher coefficient of profitability. It is equal 20 percentages. Obviously that grow of profitability rise the stability of FP. In this case only when coefficient q=1.1 FP is instable. But lifetime is signally elongated.

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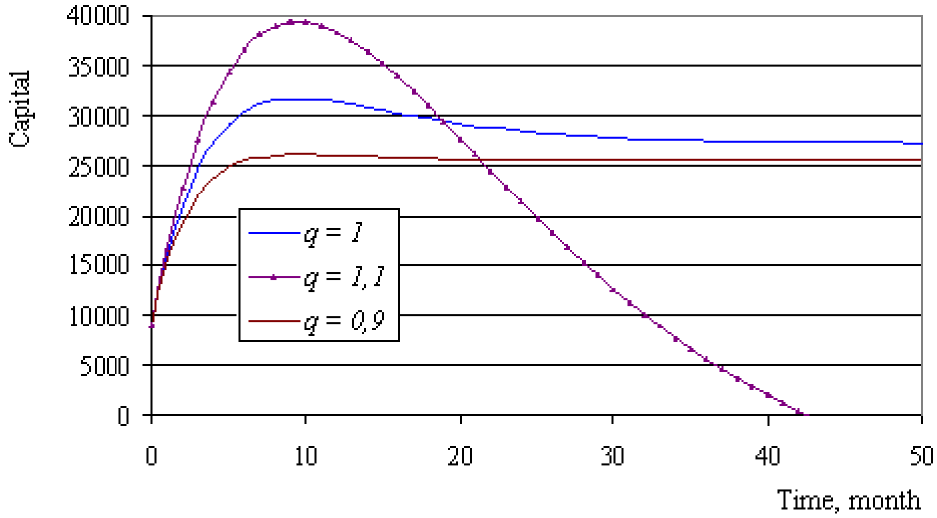

Fig. 5. **The dependence of future value of accumulated capital from the time, when** ***a=1; r=1.2; n=50; β=10%***

When profitability grows FP becomes completely stable. We can see it in figure 6.

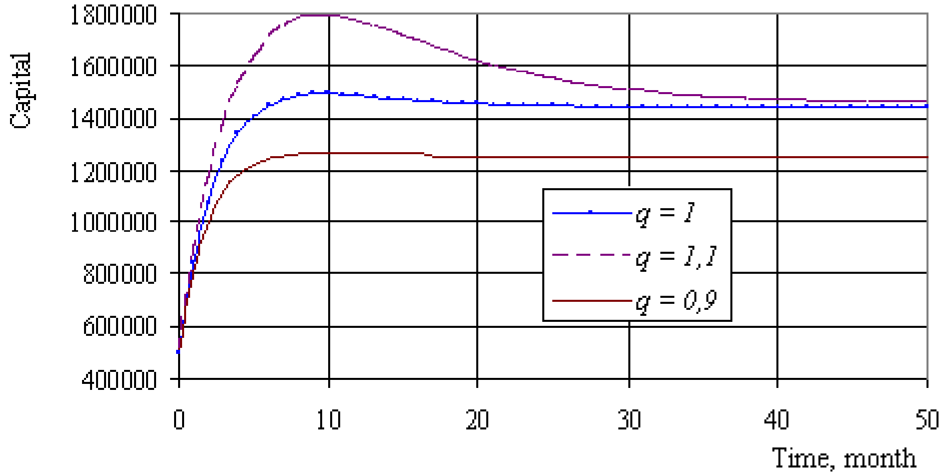

Fig.6. **The dependence of future value of accumulated capital from the time, when** ***a=1; r=1.3; n=50; β=10%***

Here in all cases we saw stability of FP. It works stable and looses features which are characteristic to FP. So it could be as normal investing company. We must mark that when number of contributions declines then stability of system grows.

Conclusions

Financial structure that use money flows for cheating purpose are called financial pyramids (FP). Information technologies let to make theoretical analysis of FP using mathematical models. Money flow modelling and research of these systems allow identify signs of such instability. We can make the following conclusions:

1. Accumulated Capital in the FP is made of 2 phases. The first is growing of capital, other declining of capital.
2. Accumulated Capital in the FP which members of money flows are fixed has equal term phases.
3. Accumulated capital in the FP which members of money flows are not fixed the second phase is variable. This phase is longer when intensity of accumulation declines and conversely.
4. When the coefficient of profitability of contribution grows and number of contribution like as interest rate declines then FP works normally like investing company.
5. Mathematical models of FP it is possible to restructure into investment models and to examine them.

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ISBN 9984 – 779 – 06 – 8