GRANULAR-INFORMATION-BASED RISK ANALYSIS IN UNCERTAIN SITUATIONS

Granulārās informācijas izmantošana riska analīzē neskaidrās situācijās

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Abstract

In the real life almost all of the decisions that we have to make incorporate uncertainty about the future events. Assessment of the uncertainty and, thus, the risk that is inherent in these decisions models can be critical. It is even truer if we are talking about the possibility of negative impact on the environment. It is very important to assess all the environmental risks in a project if there is any hazard to the environment.

In this paper the possibility of using granular information is considered. The main advantage of the granular information is that it can be used to assess risks in situations when information about future events is incomplete and imprecise. Moreover, we can use natural language to describe the problem area, as granular information paradigm uses both fuzzy and probabilistic information.

We propose to use entropy as the measure of uncertainty. However, the definition of entropy should be generalised, as values of probabilities, upon which the calculation of entropy is based on, are interval-valued. We propose several possibilities of generalizing the definition of entropy. Furthermore, we analyse these approaches to see whether the additivity feature holds for the generalized entropy.

Keywords: *risk analysis, fuzzy logic, f-granules, reasoning under uncertainty, entropy.*

Introduction

We live in the world where uncertainty is inherent in the vast majority of decisions that we have to make. It is no surprise that uncertainty prevails decisions, which are made in almost every field of human activity, from deciding whether to take an umbrella or not when going outside to managerial and political decisions, especially in a country like Latvia, which is in its transition period. This requires the development of more robust decision models. There are several analytical approaches to deal with uncertainty. This paper proposes to use granular bodies of evidence to model uncertainty of our world. One of the advantages of granular information is that it incorporates fuzzy information and probabilities, which also can be represented with the help of fuzzy values. Moreover, these paradigms enable one to use natural language to describe the problem, which facilitates modelling of the decision. However, generally fuzzy logic and probability theory are not used together to deal with uncertainty.

We show some of the advantages of using two of these paradigms together and propose a method for risk analysis based on this model.

Generally speaking, uncertain events can be considered as opportunities or risks, depending of whether they turn out to be favourable or not. The main tool for dealing with uncertainties is risk analysis. In the following chapters we consider our approach in detail. In chapter 2 we give a brief description of entropy and its relation to uncertainty. In chapter 3 we review fuzzy and granular information. Chapter 4 considers the possibility of using fuzzy granules and entropy in risk assessment. Moreover, in this chapter we suggest a generalized definition of entropy and give its analysis. In chapter 5 concluding remarks are given.

Entropy – Uncertainty about the Event

The possibility of using entropy for risk assessment was proposed in [1, 2]. Entropy is a notion from information theory used to measure amount of lacking information. Thus, we can use entropy to measure uncertainty associated with each alternative. Entropy is a criterion that

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can be used by a decision maker who is adverse to uncertainty regardless of the value associated with each of the outcomes. He does not care what happens as long as he knows [1].

Usually risk is related to the uncertainty of the future events. In other words, risk is related to lack of knowledge about future events. So it would be natural to define risk as *amount* of the lacking information. Thus, in this approach we adapt the idea that risk reflects how much we do not know about the future.

Entropy is the basic notion in the information theory field. Informally we can define entropy as the measure of uncertainty of a system that at a given moment can be in one of the states, where set of all the possible states is defined and the probability that system is in some state is known.

We can define entropy formally as follows. Let us assume that there are n different states that a system can be in:

$$s_1, s_2, \dots, s_n.$$

With formula (1) we will denote that the probability that system S is in state s_i is p_i.

$$P(S \sim s_i) = p_i, \quad i = 1 \,..\, n. \tag{1}$$

If values of all p_i in (1) are known, then entropy of a system can be calculated according to (2).

$$H = -\sum_{i=1}^{n} p_i \log p_i \ . \tag{2}$$

For further information on entropy and information theory you can refer to [3].

In the following chapters we consider the idea behind fuzzy and granular information and we show how the entropy can be calculated when values of the probabilities are interval valued.

Fuzzy Information

In this section we will consider the idea behind fuzzy information. We will give just the basic details, but you are encouraged to refer to [4, 5] for further information on fuzzy sets and fuzzy logic.

The basic idea behind fuzzy logic is that in the real world one can rarely obtain numerically precise measurements, as measurement devices have only a limited precision and humans (which also can be considered as ‘measurement devices’, in a broader sense) perceive and process information, which is fuzzy rather than precise numerically. For example, we usually say "there are a lot more small cars than big cars at a parking place" or "it is likely that it will rain today", but we rarely give a precise probability value, which denotes our confidence level that it will really rain.

Thus, we can use fuzzy values and quantifiers such as "big", "small", "old", "not very young", "most", "likely" etc. to describe perceptions of the events and processes in the real world. It is obvious that fuzzy values suit better for description of the real world, than crisp values. However, we need a lot more computational power if we are using fuzzy values. Each fuzzy value is given by a fuzzy subset represented with a membership function, which shows to what extent an element belongs to this set. For example, consider membership function of the fuzzy value "big" shown in Figure 1.

As you can see in Figure 1, all values starting from 30 are "full" members of the fuzzy subset "big", but membership of values from 15 to 30 gradually increases from 0 to 1 (i.e. from non-membership to being full members).

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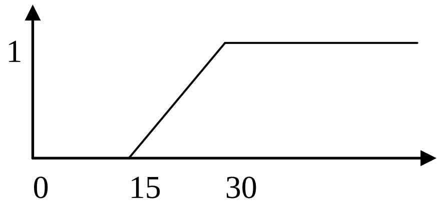

Figure 1. Fuzzy value "big"

Fuzzy logic is the basic component of the "soft computing" paradigm. In contrast to "hard" computing, we can use soft computing to do calculations when the data available is incomplete or imprecise, as well as if we do not need high degree of precision in our calculations, e.g. if they imply high costs of computing and/or modelling.

Fuzzy Granular Information

The idea of information granularity and its application in the context of fuzzy logic is presented in [6]. The idea of information granularity is very close to that of fuzzy information. Information is granular in the sense that, (a) the perceived information is fuzzy and, (b) the perceived information is granular with a granule being a bunch of values drawn together by similarity or indistinguishability.

In this paper conditioned π-granules are considered, which are characterised by propositions of the following form:

$$g_i \overset{\Delta}{=} \text{If } X = u_i \text{ then } Y \text{ is } G_i,$$

where G_i and u_i are fuzzy values and *X*, *Y* are linguistic variables. Evidence *E* can be regarded as a collection of these granules:

$$E = \{g_1, \ldots, g_N\}.$$

Variable *X* assumes its value with a specified probability. Thus, evidence is probability distribution P_X of conditional π- granules. Moreover, each granule can be regarded as conditional possibility $\Pi_{(Y|X=u_i)} = G_i$. Hence, evidence can be regarded as conditional possibility distribution $\Pi_{(Y|X)}$. Thus, evidence can be considered as the following construction:

$$E = \{P_X, \Pi_{(Y|X)}\}.$$

Given a collection of bodies of evidence $E = \{E_1, \ldots, E_K\}$, one can ask questions about the information contained in these evidences. The main question is: "what is the probability of (*Y* is *Q*)?", where *Q* is a fuzzy subset of *V*. The probability of such an event is an *interval value* in which the expected possibility $\mathrm{E\Pi}(Q)$ is regarded as the upper boundary, and the expected certainty $\mathrm{EC}(Q)$ is regarded as the lower boundary of the probability sought. In [6] the following formulae are proposed:

$$\mathrm{E\Pi}(Q) = \sum_{i,j} p_{ij} \sup\left(Q \cap G_i \cap H_j\right),$$

$$\mathrm{EC}(Q) = 1 - \mathrm{E\Pi}(Q'),$$

where *Q′* is a complement of set *Q*. These formulae are for a special case in which there are two evidences in the system but it would not present any difficulty to induce a more general form of the formula.

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In paper [7] you can find description of an adaptive network, which can be used to calculate the upper and the lower bound of the probability.

Uncertainty and Granular Information-Based Risk Assessment

As we have learnt from previous chapters, granular information can be represented with the help of conditional granules of the "IF…THEN…" form. In order to adapt this methodology to risk assessment, we can describe our problem domain with the help of such granules putting in the consequence ("THEN") part evaluation of some criterion, which is important for us and which is in direct or indirect relation to the risk inherent in our problem domain.

For example, if we are considering a project of constructing a power plant, we ought to consider to what extent it will influence the environment and the locality. Thus, we can describe our problem with a set of bodies of evidence made up of the following granules.

Evidence I, for the description of the distance to the closest inhabited area:

IF (plant is pretty close to inhabited area) with probability $p_{1,1}$ THEN influence is medium

IF (plant is close to inhabited area) with probability $p_{1,2}$ THEN influence is moderately high

IF (plant is very close to inhabited area) with probability $p_{1,3}$ THEN influence is high

Evidence II, for the description of the forecasted air pollution:

IF (air pollution is medium) with probability $p_{2,1}$ THEN influence is moderately high

IF (air pollution is not high) with probability $p_{2,2}$ THEN influence is low

Evidence III, for the description of the forecasted water pollution:

IF (water pollution is very low) with probability $p_{3,1}$ THEN influence is low

IF (water pollution is low) with probability $p_{3,2}$ THEN influence is medium

IF (water pollution is medium) with probability $p_{3,3}$ THEN influence is high

We can continue describing factors that can influence the evaluation of the chosen criterion and, as can be seen, we can incorporate our confidence in the values of these factors. Moreover, we can construct other sets of bodies of evidence, which describe evaluation of other criteria.

After we define bodies of evidence, we can calculate the probability that influence on the environment will be high or medium. Obtained probability will have an interval value $[p_{min}, p_{max}]$, which defines the lower and the upper bound of the probability.

We can calculate what are the probabilities that the criterion takes different values, e.g. "high", "low" etc. Moreover, we can consider different criteria.

In the following chapter we discuss how we can assess the uncertainty related to the probabilities obtained and how it can be related to risk.

Entropy and Interval-Valued Probabilities

Let us consider how entropy can be calculated if we are dealing with interval-valued probability values. Obviously, the entropy itself will be interval valued.

In this section we show how a generalised definition of entropy can be obtained, which is suitable for interval-valued probabilities. As before, we assume that system can be in *n* states, but the probability that system is in *i*-th state is interval and is equal to $[p_i^{min}, p_i^{max}]$. In case when probabilities are single-valued rather than interval-valued, it is required that probabilities sum to 1, i.e.

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$$\sum_i p_i = 1. \tag{3}$$

If probabilities are interval valued, then (3) can be rewritten as (4):

$$\sum_i p_i^{\min} \le 1 \le \sum_i p_i^{\max}. \tag{4}$$

It is easy to show that (3) is a special case for (4) when $p_i^{min} = p_i^{max}$ for each *i*. Moreover, if we define p_i^{avg} as (5) then it can be shown that (6) holds.

$$p_i^{avg} = \frac{p_i^{\min} + p_i^{\max}}{2}, \tag{5}$$

$$\forall i : p_i^{\min} \le p_i^{\max} \Rightarrow -p_i^{avg} \log p_i^{\min} \ge -p_i^{avg} \log p_i^{\max}. \tag{6}$$

It should be noted that states with lower probability values are more informative. Thus, we can expect that in order to calculate the upper boundary of entropy $H^{\max}$ we should use the lower probability bounds p_i^{min}. We can find the upper boundary of entropy for a system as follows:

$$H^{\max} = -\sum_{i=1}^{n} p_i^{avg} \log p_i^{\min}, \tag{7}$$

and the lower boundary of entropy:

$$H^{\min} = -\sum_{i=1}^{n} p_i^{avg} \log p_i^{\max}, \tag{8}$$

From (6) it follows that $H^{\min} \le H^{\max}$. Moreover, as was mentioned above, entropy for a system with interval-valued probability is interval-valued as well and is equal to (9).

$$H = [H^{\min}, H^{\max}]. \tag{9}$$

The obtained definition of interval-valued entropy is generalisation of the ‘traditional' entropy of a system with single-valued probabilities. In the following chapter we examine whether the additivity feature holds for the generalized version of entropy defined in (7), (8).

What about Additivity Feature?

If additivity holds, it means that if we have two independent systems, say, *X* and *Y*, then entropy of a system that is obtained by joining systems *X* and *Y* is equal to sum of individual entropies for *X* and *Y*. In other words, if additivity feature holds, then

$$H(X, Y) = H(X) + X(Y). \tag{10}$$

If we define entropy as (7) and (8), then it can be shown that if we have two systems *X* and *Y* with states accordingly $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_m$, moreover, $P(X\sim x_i)=p_i$ and $P(Y\sim y_j)=r_j$, then

$$H^{\min}(X,Y) = -\left(\sum_{i=0}^{n} p_i^{q.avg} \log p_i^{\max} + \sum_{j=0}^{m} r_j^{q.avg} \log r_j^{\max}\right) \approx H^{\min}(X) + H^{\min}(Y), \tag{11}$$

and $$H^{\max}(X,Y) = -\left(\sum_{i=0}^{n} p_i^{q.avg} \log p_i^{\min} + \sum_{j=0}^{m} r_j^{q.avg} \log r_j^{\min}\right) \approx H^{\max}(X) + H^{\max}(Y), \tag{12}$$

where $$p_i^{q.avg} = \frac{\left(p_i^{\min}\sum_{j=0}^{m} r_j^{\min} + p_i^{\max}\sum_{j=0}^{m} r_j^{\max}\right)}{2} \text{ and } r_j^{q.avg} = \frac{\left(r_j^{\min}\sum_{i=0}^{n} p_i^{\min} + r_j^{\max}\sum_{i=0}^{n} p_i^{\max}\right)}{2}. \tag{13}$$

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As can be seen, (13) does not differ from (5) much. In (13) summation factors appear. If we are dealing with single-valued probabilities then it is obvious that these sums are equal to *1* and the additivity feature holds. If the probabilities are interval-valued then from (4) it follows that formulae (14) hold.

$$0 \le \sum_{j=0}^{m} r_j^{\min} \le 1,\ \sum_{j=0}^{m} r_j^{\max} \ge 1,\ 0 \le \sum_{i=0}^{n} p_i^{\min} \le 1,\ \sum_{i=0}^{n} p_i^{\max} \ge 1\,. \tag{14}$$

Summation factors (14) can be considered as a sort of scaling factors, where the first may have reducible influence and the second may have augmenting influence, so we may expect that two of these factors compensate each other. Hence, entropy for the joined system calculated according to (11) and (12) *should not* differ much from the sum of individual entropies of the systems considered. Thus, it can be stated that we have *quasi-additivity*, as the summation factors compensate each other to some extent.

Incorporating Entropy and Risk Assessments

In the previous sections we described how we could use natural language to describe problem domain. After we construct fuzzy bodies of evidence, we can use them to calculate probability that criteria will take different values. Entropy values for systems corresponding to different criteria should be calculated separately and then summed according to the additive property of entropy. Obtained entropy is interval valued and we can use it to measure uncertainty of our problem domain and of evaluations of the criteria chosen.

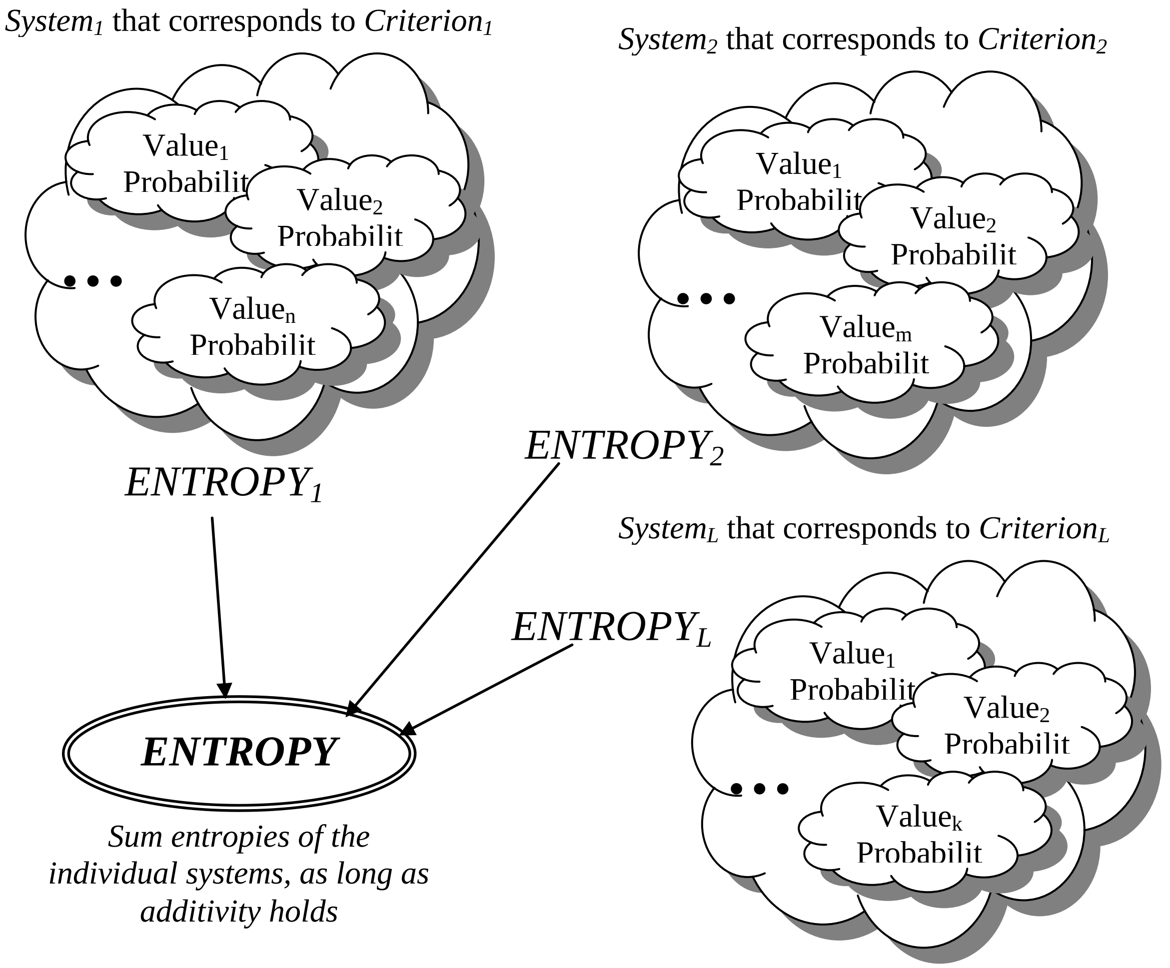

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Figure 2. Entropy calculation for an alternative

Figure 2 shows this approach towards risk assessment graphically. First we have to define what criteria are of interest for us. After that for each alternative we construct fuzzy granules that describe each criterion and evaluate the probabilities that a particular criterion will take some value. The probabilities are interval-valued. A particular value of some criterion corresponds to a separate state in a system (we use this word in a broad sense). For example, in Figure 2 the first system corresponds to $criterion_1$, which can take n different values with corresponding probabilities. After having evaluated probabilities of values for all criteria we can calculate entropy for each system. In order to get evaluation of overall entropy, we can sum entropies calculated for separate systems. Now it is clear why the additivity feature is so important. If it would not hold, we could not just sum up individual evaluations in order to get overall evaluation.

The overall entropy value obtained can be considered as evaluation of uncertainty for a particular alternative.

Conclusion

This paper shows how fuzzy granular information can be used in order to measure risk and uncertainty. The uncertainty assessment is based on the generalised definition of entropy.

One of the advantages of the approach proposed in this paper is that one can use natural language to describe problem domain, upon which the uncertainty is assessed. This is due to fuzzy logic upon which the approach is based, which enables one to use fuzzy rather than crisp values.

Moreover, we show how entropy can be generalized to the case of interval-valued probabilities and we analyse the new definition to see whether the additivity holds. We conclude that quasi-additivity holds for the generalized entropy.

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