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Using the Concept of Fuzzy Random Events in the Assessment and Analysis of Ecological Risks

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Abstract. **In many cases, the assessment and analysis of ecological risks is a complicated task, which is first of all related to obtaining reliable initial information. As a rule, ecological risks are due to unrepeated unique situations; from this it follows that sufficient statistical data on whose basis reliable evaluation of specific risks is made, are not available. On the other hand, unfavourable impacts on the external environment can affect the components of an ecosystem differently. The complexity of correlations among the components of an ecosystem significantly complicates an analysis of possible impacts on the components of a specific system.**

When statistical data are missing or insufficient, experts who perform the required assessment on the basis of their knowledge and experience but often also using their intuition, are the only source of initial data. Here, however, the problem of reliability of expert evaluations arises. If other sources of information are missing, we have to accept subjective evaluations of experts as a basis, without an opportunity to evaluate the degree of their confidence.

In this kind of situation, it seems to be validated to introduce the extent of uncertainty into the evaluations of parameters of ecological risks. This can be accomplished by using fuzzy initial evaluations. This paper focuses on the concept of fuzzy random events and shows favourable chances of using that concept in the assessment and analysis of ecological risks.

Keywords: **Function of membership to a fuzzy random event, fuzzy random event, operations on fuzzy events, probability distribution function, probability of a fuzzy random event.**

I INTRODUCTION

Any risk can be characterized by two components (dimensions): [1] potential losses that might occur due to the presence of unfavourable circumstances; (2) chances of occurrence of unfavourable circumstances (factors) which are commonly specified by means of probabilistic evaluations.

Depending on the character of unfavourable circumstances (factors) and related to them potential losses, risks can be classified as economic, political, social, military etc. Quite a wide group is composed of ecological risks. Those risks can be roughly divided into risks for economic activity of humans due to inefficient use of natural resources (flooding of large areas caused by building power stations, territory degradation due to the felling of the forests and predatory usage of lands). Another field of ecological risks is related to harmful impacts on an environment as a result of human economic activity. This kind of risk occurs mainly due to harmful extras that negatively affect the components of ecosystems.

To assess any risk, its two components have to be assessed: losses that might occur as a result of one of another human activity or effect of unfavourable external and internal factors, and probabilities of those losses. When sufficient statistical data are available, there is no difficulty to assess the risks. A visual example of this is the assessment of risks of fire and car accident performed by insurance companies. Evaluations of such risks are obtained on the basis of statistical material; due to that, their reliability is very high. Quite a different situation is with assessing unique ecological risks. Here statistical data are not available, as a rule. The only source of information is specialists - experts who accomplish requested assessment on the basis of their knowledge and experience but sometime –intuition. It is impossible to evaluate the reliability of expert evaluations a priori since the results of expert evaluations can be heavily affected by different heuristics and nonobjectivities. A detailed analysis of such heuristics and nonobjectivities is provided in [4]. There are also proposed some techniques that enable one to a priori evaluate potential biases that may have place in the evaluations of some experts. By means of such techniques it is only possible to slightly correct evaluations of experts under the possibility of

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potential common mistakes, however total reliability of those evaluations remains at the sign of question.

Recently, techniques have been developing intensively that enable obtaining and using uncertain expert evaluations. A smaller or greater part of uncertainty is artificially incorporated into the initial evaluations. Otherwise, instead of one deterministic evaluation, say, the probability of random event occurrence, a generalized uncertain evaluation is integrated which includes a set of deterministic evaluations with different degrees of uncertainty. The obtained uncertain evaluations are processed and aggregated; as a result, uncertain resulting evaluations are derived. This kind of approach seems to be more preferable as compared to the use of deterministic initial and resulting evaluations regarding which there is no any confidence that they adequately represent the existing state of things.

To model the uncertainty of expert evaluations, fuzzy set theory is widely used. This common approach can be divided into two major directions: (1) using the concept of fuzzy random events and (2) using fuzzy evaluations of the probabilities and losses. This paper discusses the concept fuzzy random events, defines probability calculations for such events and analyses the possibility of using this kind of event in the assessment and analysis of ecological risks.

II WHAT IS A FUZZY RANDOM EVENT?

Let us define the notion of a fuzzy random event using a simple and visual example. Let us assume that an accident may occur at the chemical plant, as a result of which a harmful substance can get into an external environment. That substance might cause a substantial damage to some components of the surrounding ecosystem.

To assess the risks related to the potential accident at the plant, it is necessary to assess probabilities of different level of environment pollution. For that purpose, let us introduce a nondimensional factor of pollution extent:

$$c = \frac{c_r}{c_{mpc}} \tag{1}$$

where c_r - real concentration of harmful substance in the environment; c_{mpc} - maximum allowable concentration of that substance.

Since c is a continuous random variable, probability distribution function $F(c)$ can be constructed for it. When statistical data are not available, the construction of the requested distribution function can be made on the basis of expert evaluations. Methods of construction of subjective distribution functions are considered in detail in [6], [7]. Not going into technical details, let us assume that based on the data provided by an experienced expert, distribution function $F(c)$ is constructed for the continuous random variable c defined in expression (1). A graph of that distribution function is shown in Fig.1.

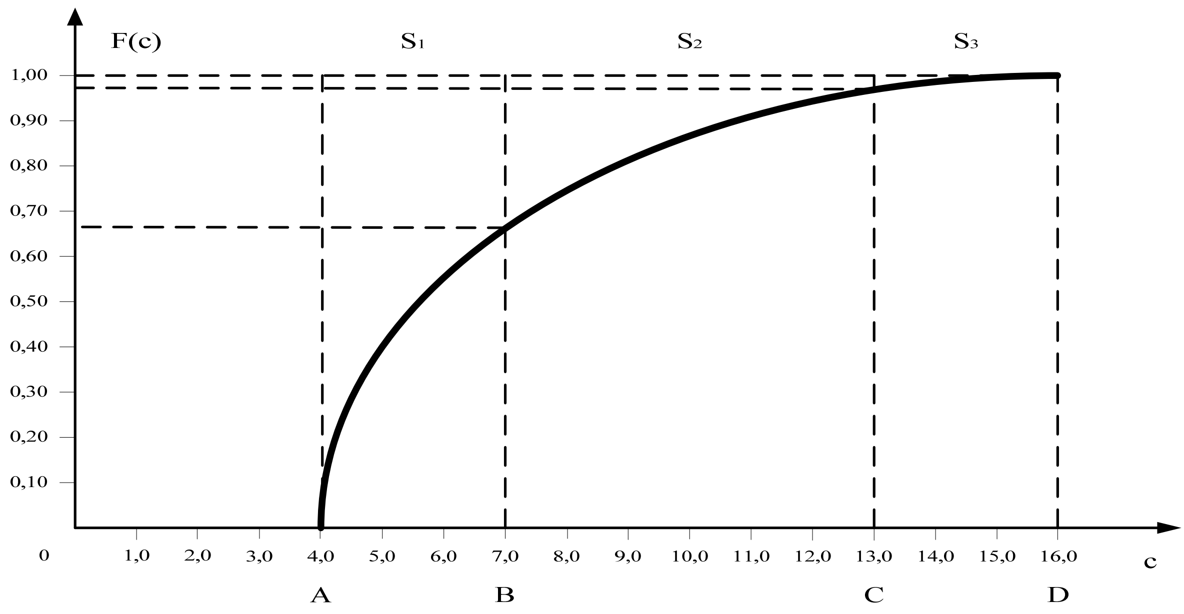

Fig.1. A graph of distribution function $F(c)$, constructed using expert evaluations of an expert

For further analysis it is worth distinguishing some generalised categories of pollution of the environment. In Fig.1 these categories are depicted as S_1, S_2, S_3 and can be interpreted as follows:

S_1 - low pollution: $c \in [4,0;7,0]$;

S_2 - moderate pollution: $c \in [7,0;13,0]$;

S_3 - heavy pollution: $c \in [13,0;16,0]$.

From the graph of distribution function $F(c)$ shown in Fig.1 it is easy to determine probabilities of each of pollution categories:

$$p(S_1) = F(B) - F(A) = 0,65 - 0,00 = 0,65;$$

$$p(S_2) = F(C) - F(B) = 0,97 - 0,67 = 0,30;$$

$$p(S_3) = F(D) - F(C) = 1,00 - 0,95 = 0,05.$$

The problem with the above-mentioned separation of pollution level into categories is related to the determination and interpretation of borders between the categories. Let us assume that to lessen the risk, specific actions are required for each category. Let us also assume that for a certain scenario of possible development of the situation a conclusion is made that potential pollution is evaluated as $c = 0,66$, but for alternative scenario an evaluation of the pollution level $c = 0,74$ is obtained. Hence, the planned actions aimed at lessening the risk, formally have to be different for each scenario, which seems to be improbable taking into account that the difference between evaluations $c(.)$ is insignificant.

In this kind of situation it is worth to somehow „wash away" the borders between the categories. A correct technique for that purpose is to use fuzzy categories instead of unambigously specified ones. One possible way to represent the aforementioned categories in a fuzzy form is depicted in Fig. 2.

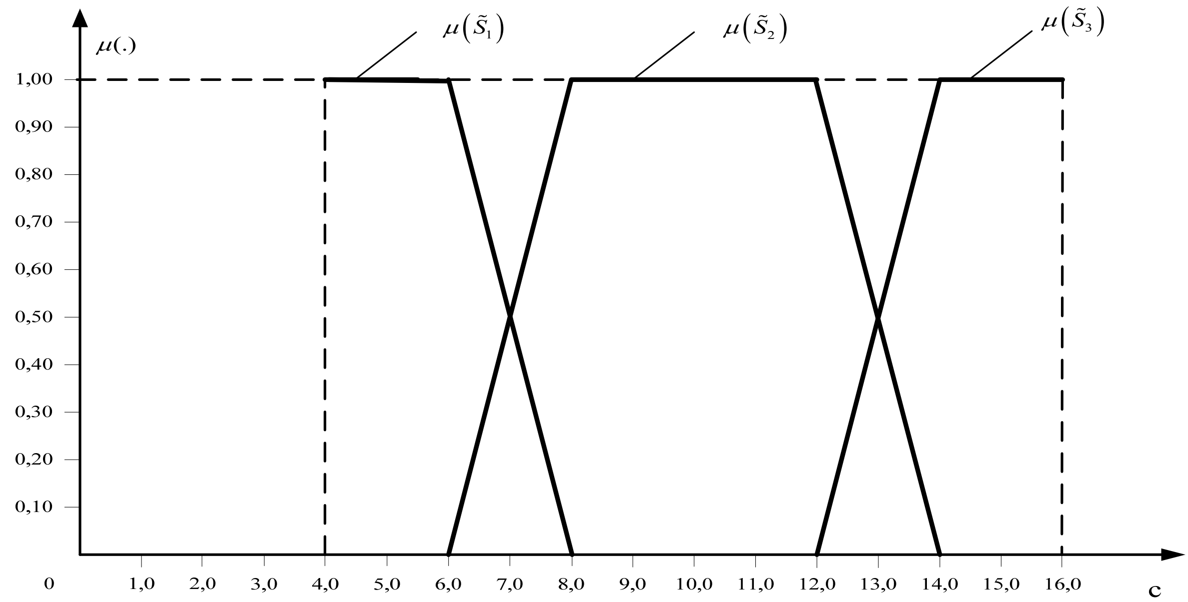

Fig.2. Representation of deterministic categories of environment pollution level in a fuzzy form

The falling of real pollution level into one of categories S_1, S_2, S_3 in Fig.1 can be interpreted as a deterministic random event. Then the falling of real level of pollution into one of fuzzy categories $\tilde{S}_1$, $\tilde{S}_2$, $\tilde{S}_3$ in Fig.2 can be viewed of as fuzzy random event: real level of pollution falls into a fuzzy category $\tilde{S}_i$ with the membership degree $\mu(\tilde{S}_i)$, $i = 1,2,3$.

Let us then assume that the forecasted level of environment pollution is characterised by value $c = 0,65$. Using graphs of membership functions $\mu(\tilde{S}_1)$ and $\mu(\tilde{S}_2)$ it is easy to determine that this pollution level with the membership degree $\mu(\tilde{S}_1) = 0,73$ belongs to the fuzzy category $\tilde{S}_1$, but with the membership degree $\mu(\tilde{S}_2) = 0,27$ belongs to the fuzzy category $\tilde{S}_2$. This is the principal difference between deterministic categories and fuzzy categories. In the first case, real value of the relevant random variable may only belong to a single category, buti n the last case – to several fuzzy categories.

How the fuzzy borders between the categories are formed? In the interval $c \in [4,0;6,0]$ pollution is unambigously related to category S_1, while in the interval $c \in [6,0;8,0]$ the pollution may be either ascribed to category S_1, or – category S_2 with different degree of membership. In the interval $c \in [8,0;12,0]$ pollution is unambigously related to category S_2. Pollutions in the intervals $c \in [12,0;14,0]$ and $c \in [14,0;16,0]$ can be interpreted in similar way.

Let us briefly consider the theory of fuzzy events. The notion of a fuzzy event and its probability was first stated by L. Zadeh in his underlying work [10].

Definition. Let (R^n, Θ, P) be a probabilistic space, in which Θ is σ-field of Borel sets in R^n, but P is the probabilistic evaluation in R^n. Then *a fuzzy event* in R^n is a fuzzy set A in R^n, whose membership function, μ_A $(\mu_A : R^n \to [0,1])$, is Borel-measurable

If a fuzzy event $\tilde{A}$ is related to a continuous probabilistic function in the domain of definition of $\tilde{A}$, the probability of this event can be calculated as follows [10]:

$$P(\tilde{A}) = \int_{R^n} \mu_{\tilde{A}}(x) dP = E(\mu_{\tilde{A}}). \qquad (2)$$

Strict mathematical details of fuzzy event theory can be found in [2].

Fuzzy events are in essence fuzzy subsets specified in the domain of definition of relevant random variable. Say, in the example considered above fuzzy events $\tilde{S}_1$, $\tilde{S}_2$ and $\tilde{S}_3$ are typical fuzzy subsets with trapezoidal membership functions $\mu(\tilde{S}_i)$, $i = 1,2,3$; from this it follows that all operations on fuzzy sets can be applied to fuzzy events. More detailed information about the operations on fuzzy sets is provided in [3], [5] and [8].

Let us calculate the probabilities of the considered above fuzzy categories using expression (2).

$$P(\tilde{S}_1) = \int_{4,0}^{6,0} \mu_{\tilde{S}_1}(c) dP + \int_{6,0}^{8,0} \mu_{\tilde{S}_1}(c) dP = (F(6) - F(4)) + \frac{1}{2}(F(8) - F(6)) =$$

$$= (0,55 - 0) + \frac{1}{2} * (0,75 - 0,55) = 0,55 + \frac{1}{2} 0,20 = 0,65 .$$

Here $F(.)$ is the value of distribution function at the corresponding value of c.

$$P(\tilde{S}_2) = \int_{6,0}^{8,0} \mu_{\tilde{S}_2}(c) dP + \int_{8,0}^{12,0} \mu_{\tilde{S}_2}(c) dP + \int_{12,0}^{14,0} \mu_{\tilde{S}_2}(c) dP =$$

$$= \frac{1}{2}(F(8) - F(6)) + (F(12) - F(8)) + \frac{1}{2}(F(14) - F(12)) =$$

$$= \frac{1}{2}(0,75 - 0,55) + (0,95 - 0,75) + \frac{1}{2}(0,98 - 0,95) = \frac{1}{2} * 0,20 + 0,30 + \frac{1}{2} * 0,03 = 0,315 ;$$

$$P(\tilde{S}_3) = \frac{1}{2} \int_{12,0}^{4,0} \mu_{\tilde{S}_3}(c) dP + \int_{14,0}^{16,0} \mu_{\tilde{S}_3}(c) dP = \frac{1}{2}(F(14) - F(12)) + (F(16) - F(14)) =$$

$$= \frac{1}{2}(0,98 - 0,95) + (1,00 - 0,98) = \frac{1}{2} * 0,015 + 0,02 = 0,035 .$$

It is easy to see that the sum of probabilities of all fuzzy events is equal to 1. This is due to the fact that for all distribution functions whose graphs are shown in Fig.2 the following requirement holds

$$\sum_i \mu_{\tilde{S}_i}(c) = 1 \qquad (3)$$

for all values $c \in [4,0;16,0]$.

It should be noted that in the fundamental work [10] condition (3) has not been formulated as an obligatory requirement; the requirement was formulated later so as to ensure that the values of probabilities of fuzzy events are similar to the values of common probabilities for the complete group of events. (for more details see [4]). If membership functions of relevant fuzzy events are constructed so that condition (3) is not satisfied, the resulting values of probabilities can be renormated properly to ensure that condition (3) holds.

III RESULTS AND DISCUSSION

In the example discussed above, both at deterministic and fuzzy borders between the categories, category *„low pollution"* has the highest probability of occurrence but category „heavy pollution" has the lowest probability of occurrence. In both cases it is only related to the shape of graph of distribution function $F(c)$ in Fig.1.

It is not difficult to calculate the values of probabilities of fuzzy events $P\left(\tilde{S}_i\right), i=1,2,3$ by expression (2) because graphs of membership functions $\mu\left(\tilde{S}_i\right), i=1,2,3$ in Fig.2 have the trapezoidal shape and operations of integration are extremely simple. In real tasks, graphs of functions of membership in fuzzy events (categories) are, as a rule, of trapezoidal or triangular shape, which sufficiently simplifies calculation of probabilities of respective fuzzy events. In case if the graphs of membership functions are of different shape, they have to be represented in analytical form; for that purpose approximation may be used.

Where the concept of fuzzy random events can be used? A good example of such application is discussed in [9], where the author solves the task of modelling and practical evaluation of processes in real fuzzy stochastic system. Another widely used application of that concept is tasks of fuzzy classification of the objects. The theoretical grounds of solving such tasks are considered in [1].

The concept of fuzzy random events can also be successfully applied in the case of discrete random variables. If some statistical data about realizations of random variable X in the past are available, deterministic values of probabilities of falling the values of this variable into the specified fuzzy categories can be evaluated. For the calculation of probabilities of fuzzy random events (of falling the next realisation of X in the corresponding fuzzy categories), expression (2) is transformed as follows:

$$P\left(\tilde{A}_i\right)=\sum_{x_j \in \tilde{A}} p\left(x_j\right)\mu_{\tilde{A}_i}\left(x_j\right), i=1,\ldots,n. \quad (4)$$

What are potential advantages of using the concept of fuzzy random events in the assessment and analysis of ecological risks? First, multiple studies have shown that humans better think in terms of fuzzy categories, i.e., they better express their judgements and evaluations in a fuzzy environment. Second, the specifics of ecological risk assessment in most of cases is that the initial data are either absent or insufficient to employ standard statistical techniques. Effective modelling of the uncertainty of results of expert evaluations and further use of such uncertain information allow one to more successfully assess and analyze various ecological risks.

IV CONCLUSIONS

This work analyses possibilities of using the concept of fuzzy random events in the assessment and analysis of ecological risks. Taking into account intensive development of industry, problems of risk assessment and prevention from negative impact on the environment are becoming ever more important both in the developed and developing countries. The lack of proper objective data makes it necessary to widely use expert evaluations; however the requirement of deterministic evaluations from experts is in evident contradiction with the requirement of reliability of such evaluations. Due to that, it seems necessary to use uncertain expert evaluations. The modelling of expert evaluation uncertainty can be performed in different ways. The technique discussed in this paper, i.e., using fuzzy random events, exemplifies one of possible directions. It can be successfully employed for practical evaluation and analysis of different types of ecological risks.

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