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RIGIDITY OF RUBBER-METAL ELEMENTS WITH THIN LAYERS AT COMPRESSION

ELASTOMĒRA AMORTIZATORA STINGUMA APRĒĶINS, ŅEMOT VĒRĀ TĀ STARPSLĀŅA DEFORMĀCIJU

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Abstract: *There is described a method of generation the rigid feature "Force – Settlement" for thin-layer rubber-metal compensating elements, which consist of several rubber and non-elastomeric layers, operating when being pressed, taking into account the low compressibility of rubber layers and deformation of support non-elastomeric layers. Variational method of theory of elasticity for compressible materials is used. It is recommended to use the acquired analytic dependencies when analysing the element and designing multi-layer compensating elements, as well as when determining the value of Poisson coefficient for rubber-like materials.*

Keywords: *rubber, shock-absorber, rigidity, weak compressibility.*

Introduction

The multi-layer thin-layer rubber-metal shock-absorbing elements are widely used in various fields of mechanical and civil engineering ($\rho = a/h >> 10$, a – is a typical geometrical dimension in the design project; h – is a width of rubber layer), and have lots of structural advantages, in particular, they ensure greater rigidity under axial compression and lower rigidity under shift and spinning. In such constructions very thin metal layers are used as supporting intermediate layers, to which rubber layers are attached by vulcanization.

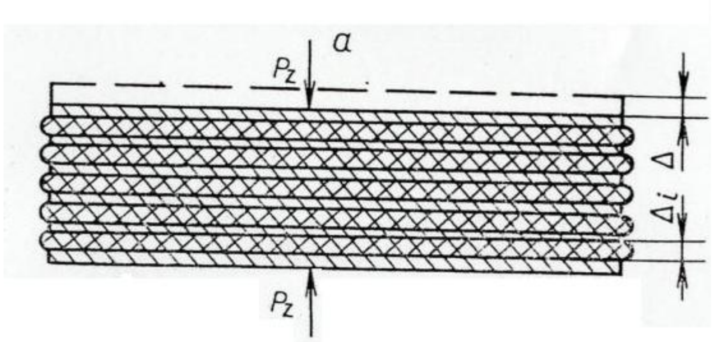
The calculations of rigid dependencies (of the type "Force - Settlement") for such thin-layer metal elements being pressed and by using already classic solutions [1 – 4], showed that there is inherent difference between the calculated values and the experimental data, when layers of works [5, 6] have certain geometrical dimensions. At the same time, the closer Poisson coefficient is to 0.5 and the thinner are rubber and supporting layers in compensating elements, the greater is the divergence. The result of experimental researches [5, 6] of such elements being compressed is that the rigid feature of multi-layer compensating elements is considerably influenced by: low compressibility of rubber material, especially, when Poisson coefficient of rubber changes within 0,480 ÷ 0,499; deformation of supporting layers being sufficiently thin.

To ensure safe performance and when designing thin-layer rubber-metal compensating devices, it is necessary to obtain analytical dependence for rigid feature "Force - Settlement" of such kind of elements. It is only possible with the correct design model, which will let us take into account and estimate all the geometrical parameters and physical and mechanical features of materials of the considered elements.

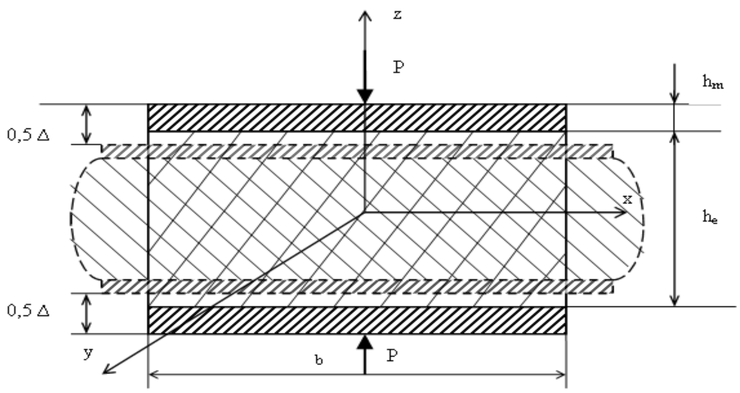
In this work is proposed one of the design models for analytical calculation of rigid feature of multi-layer compensating elements being compressed, which lets take into account the low compressibility of material of rubber layers and deformation of non-elastomeric supporting layers.

Materials and methods

The method of obtaining analytical dependence "Force - Settlement" of multi-layer compensating element under axial compression is considered. Only small deformations are considered. Application of the proposed method is shown by the example of designing multi-layer shock-absorber, which consists of thin flat rectangular elements.
The geometrical design model is shown in Figure 1 (a) and (b).



(a)



(b)

Fig.1. **Model of calculation: (a) - the shock absorber; (b) – n^{th} - layer of shock absorber**

Proposed method is using the variational method, which is based on the principle of minimal potential energy [1, 8] for low compressible material. The potential energy of the studied element in case of small deformations is written this way:

$$J = \Sigma_{l=1}^{l=n} G \int_V \begin{bmatrix} (\varepsilon^2{}_{xx} + \varepsilon^2{}_{yy} + \varepsilon^2{}_{zz} + 2(\varepsilon^2{}_{xz} + \varepsilon^2{}_{yz} + \varepsilon^2{}_{xy}) + \\ + \dfrac{3\mu}{1+\mu} S(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) - \dfrac{9\mu(1-2\mu)}{4(1+\mu)^2} S^2 \end{bmatrix} \bullet dV - P\Delta \tag{1}$$

where: G – modulus of rigidity for each layer;
μ – Poisson coefficient of material of each rubber layer;
P – longitudinal force of compression;
Δ – settlement of the entire element;
s – hydrostatic pressure function in each layer;
u, v, w – displacements of randomly chosen point in each layer of the layer, respectively, in directions x, y, z;
V – volume of each layer.
The summing up is carried out for all rubber and non-elastomeric layers of a multi-layer element.
Deformations ε_{ij} in each layer are found using formula:

$$\varepsilon_{xx}=\frac{du}{dx};\quad \varepsilon_{yy}=\frac{dv}{dy};\ \varepsilon_{zz}=\frac{dw}{dz};$$

$$\varepsilon_{xy}=\frac{1}{2}(\frac{du}{dy}+\frac{dv}{dx});\ \varepsilon_{yz}=\frac{1}{2}(\frac{dv}{dz}+\frac{dw}{dy});\ \varepsilon_{zx}=\frac{1}{2}(\frac{du}{dz}+\frac{dw}{dx}); \qquad (2)$$

The potential energy for the entire element is calculated by summing up formula (1) for all rubber and non-elastomeric layers. Physical and mechanical features of the material layers and geometrical parameters of layers have such indexes: e – for rubber layers; m – for non-elastomeric layers.

In order to use functional (1) when choosing functions of displacements u (x, y, z), v (x, y, z), w (x, y, z) and functions of hydrostatic pressure s (x, y, z), it is enough to fulfill geometrical boundary conditions and the conditions of coupling rubber and non-elastomeric layers for displacement functions. For simplicity let us suppose all the layers have same dimensions in the design (a and b), all the rubber layers have width h_e, and the non-elastomeric layers have width $h_м$. For the considered problem the necessary geometrical conditions are:

$$w_e(x,y,0.5h_e) = -0.5\Delta;\ \ w_e(x,y,-0.5h_e) = 0.5\Delta$$

$$u_e(x,y,\pm 0.5h_e) = u_м(x,y,\pm 0.5h_e);\ \ v_e(x,y,\pm 0.5h_e) = v_м(x,y,\pm 0.5h_e) \qquad (3)$$

When writing displacement functions analytically let us suppose that: for rubber layers the hypothesis of plane sections is true; for non-elastomeric layers the condition of homogeneous deformation is fulfilled. In this case, taking into account the geometrical conditions (3), the desired displacement functions can be chosen in the form for:

- rubber layers:

$$u_e = C_1 x (z^2 - h_e^2/4) + K_1 x,\qquad v_e = C_2 y (z^2 - h_e^2/4) + K_2 y,$$

$$w_e = - C_3(z^3/3 - zh_e^2/4)/h_e^3 - C_4 z,\quad s_e = C_5(z^2 - h_e^2/4),$$

- non-elastomeric layers:

$$u_м = K_1 x,\ V_м = K_2 y,\qquad w_м = s_м = 0, \qquad (4)$$

where: C_1, C_2, C_3, C_1, C_2, K_1, K_2 – are unknown constants, which can be found using the settlement of the element Δ from the minimum condition of full potential energy of deformation (1) of the entire element:

$$\frac{\partial J(C_1,C_2,C_3,C_4,K_1,K_2)}{\partial(C_1,C_2,C_3,C_4,K_1,K)}=0 \qquad (5)$$

Δ – the desired unknown settlement of the element, which, by using equations (3) – (5), can be found from the equation:

$$\Delta = - C_3 h_e^3/6 + C_4 h_e \qquad (6)$$

From algebraic equation system (5) and (6) for the considered element the desired dependence "Force – Settlement" can be written as:

$$\Delta=\frac{P\,h_e n}{2{,}5\,G_e ab}\,\frac{1+1.25\dfrac{B_1B_2}{\chi(B_1+B_2)}}{1+\dfrac{B_1\,B_2}{B_1+B_2+\dfrac{1-2\mu}{\mu}B_1\,B_2}} \qquad (7)$$

where:

$$B_1=1+\frac{5\,\alpha^2}{12};\ B_2=1+\frac{5\,\beta^2}{12}$$

$$\alpha=\frac{a}{h_e},\quad \beta=\frac{b}{h_e},\ \chi=\frac{G_m h_m}{G_e h_e} \qquad (8)$$

a, b, h_e, h_m – are geometrical parameters of flat rectangular rubber and non-elastomeric layers;
G_e, G_m – modulus of rigidity of material, respectively, of rubber and non-elastomeric layers;
n – number of rubber layers in the packet.

If rubber and non-elastomeric layers have different dimensions, which let us ignore the low compressibility of rubber layers and flexibility ($h_e < h_m$, $G_e << G_m$, t.i. parameter $\chi \to \infty$) of non-elastomeric layers, then from formula (7) we obtain dependence for element settlement:

$$\Delta_0 = \frac{Ph\ n}{2{,}5\,G\ ab}\,\frac{1}{1+\dfrac{\alpha^2\beta^2}{\alpha^2+\beta^2}}\,, \tag{9}$$

which coincide with the dependence “Force - Settlement”, obtained in work [1] without taking into account the compressibility of rubber and deformation of non-elastomeric layers.

Results

From formula (7) it follows that neglect of deformation of non-elastomeric layers when determining settlement of the element may lead to significant quantitative errors. As an example let us consider the element of such geometry:

$a = b = 8$ cm, $h_e = 0{,}2$ cm, $G_e = 10$ kg/cm^2, $h_M = 0{,}02$ cm, $G_M = 2{,}8\text{x}10^5$ kg/cm^2,

From (7) for the desired settlement Δ of the element and taking into account deformation of non-elastomeric layers we obtain the expression:

$$\Delta = [1+1.25\frac{B_1B_2}{\chi(B_1+B_2)}]\ \Delta^* = 1{,}298\ \Delta^* \tag{10}$$

where:

Δ^* - is the settlement of the element (see (11)) only taking into account low compressibility of rubber layer and neglecting deformation of non-elastomeric layers. It is obtained in [1; 8] and is the particular case of formula (7).

The numerical values are quite well described by formula (10) for the experimental results of work [6]. In the considered example neglect of deformation of non-elastomeric layers leads to underrating the value of element settlement approximately per 30%.

If the geometry of thin-layer element is such that it is possible to neglect only flexibility of non-elastomeric layers, then from formula (7) for settlement of element follows the dependence:

$$\Delta^* = \frac{P\,h\,n}{2{,}5\,G\ ab}\,\frac{1}{1+\dfrac{B_1\,B_2}{B_1+B_2+\dfrac{1-2\mu}{\mu}B_1\,B_2}} \tag{11}$$

it can be recommended, using the results of experiment under axial compression for multi-layer element, with quite rigid non-elastomeric layers, to obtain Poisson coefficient of rubber material. This problem for low compressible material formulated in [5] requires complicated experimental technique, and application of formula (11) lets us use quite simple experimental investigations.

Conclusion

A method proposed for determination of rigidity dependence "Force - Settlement" for multi-layer shock-absorbing elements being under pressure, and it lets take into account low compressibility of material of rubber layers and deformation of non-elastomeric layers. The considered method is quite simple in use for thin-layer elements of any configuration. The proposed method makes more thorough analysis of ready elements and lets us make decisions on optimal design of thin-layer shock-absorbing elements more effectively.

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