Mathematical Modeling of the Sequence of Machining Sections of Complex Surfaces when Milling on a Triaxial CNC Machine Tool

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Abstract. The idle running times of the working units of a machine tool are the sum of the idle running times for the tool change and for changing the area under treatment. The paper presents mathematical models, establishing the relationship between the additional time for performing the technological operations with the parameters of both the technological equipment and the object under treatment. The mathematical models for minimizing the idle moves when a tool passes from one machined section to another, allows to reduce the additional treatment time, which, in turn, leads to an increase in the productivity of the process of actual milling.

Keywords: pure/actual/finish milling, complex surfaces, CNC machine tool, optimization.

I. INTRODUCTION

The idle moves of the working modules of a machine are the sum of the idle moves for cutter change and the idle moves for changing the machined area. The time, spent on the tool idle running is the additional part of the machine operating time. In case of finish mechanical machining of complex surfaces and of significant number of machined sections, to which, as a rule, the complex surface is broken, a great number of milling cutters is utilized. This results in increasing the additional, i.e., the auxiliary operating time. The reduction of the auxiliary time when machining details significantly increases the efficiency of the used technological equipment.

Hence, increasing the productivity of machining on triaxial CNC milling machines can be achieved, on the one hand, by reducing the cutting time, and, on the other hand, by reducing the time, spent on performing idle moves.

Reduction of the time for performing idle tool moves can be achieved by minimizing the idle moves when changing the machined sections and by optimizing the machining sequence of the individual sections.

This work studies the possibility of finding an optimal sequence of performing the transitions during all types of idle tool moves at minimum time, spent on their execution.

II. MATERIALS AND METHODS

An algorithm for optimization of the cutting tool idle moves is presented in [1, 2, 3, 4, 5].

The mathematical model [1], developed by the authors, allows to determine the sequence of performing transitions under the conditions of: - minimizing the length of the trajectory of the tool's idle moves when passing from one given point to another; - complying with various constraints (geometry of both the machined workpiece and the fixing device, constraints, related to the dimensions of the machine working area etc.).

The authors claim that as a result of the practical implementation of the algorithm for constructing the path of the idle tool moves on multi-operational lathes when processing workpieces with a length-to-diameter ratio of less than 0.5, reduction of the auxiliary time by 20-30% is achieved, and the quality of the processed surface corresponds to the set quality.

The task of optimizing the general machining strategy is described in detail in paper [1]. The proposed algorithm provides the search for an optimal sequence of performing the transitions on multi-operational machines.
and their idle tool moves, at minimum expenditure of time on their execution.

When developing the algorithm, however, the authors do not take into account the tool life factor, on which the number of the cutters, used for replacement, depends, and which ultimately affects the time for performing the idle moves. The problem of minimizing the idle tool moves is solved for the case, in which the positions of the starting and the ending point of machining a section coincide. The algorithm, proposed by the authors, is intended only for a certain type of equipment, and more precisely for CNC lathes. For milling machines, in which the positions of the starting and the ending point of machining do not coincide, the application of the proposed algorithm is impossible.

In the process of mechanical machining on triaxial milling machines, the length of the idle move from one section to the next is not equal to the distance, traveled in the opposite direction, due to the mismatch of the starting and the ending point and the withdrawal of the tool, as well as due to the geometric features of the workpiece surface. Considering this, the task to determine the optimal sequence of machining individual sections can be correlated to a logistic transportation problem, namely, the Asymmetric capacitated vehicle routing problem (ACVRP). ACVRP is a combinatorial optimization and linear programming problem, in which, for vehicles of the same load capacity, located at a depot, the minimum by cost (money, time or distance) closed routes must be found, which would fully allow for servicing all customers (Fig. 1).

At the same time, the limiting load capacity condition for the means of transport must be met for each of the routes (besides, the length of the route from A to B is not equal to the length of the route from B to A). [6].

The similarity between the problem of determining the optimal sequence of transitions from one section to another and the ACVRP is presented below as a table of analogies.

<table>
<thead>
<tr>
<th>Asymmetric capacitated vehicle routing problem</th>
<th>Problem of determining the optimal sequence of tool transitions from one section to the next</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depot; Tool change position</td>
<td>Tool magazine;</td>
</tr>
<tr>
<td>A fleet of motor vehicles</td>
<td>Section</td>
</tr>
<tr>
<td>Customer</td>
<td>Tool</td>
</tr>
<tr>
<td>Automobile; Tool load capacity</td>
<td>Tool life period</td>
</tr>
<tr>
<td>Load requested by one customer</td>
<td>Needed cutting time for one section</td>
</tr>
</tbody>
</table>

The algorithm of Clark-Wright, consisting in merging small routes into larger ones until it becomes possible to reduce the total cost of the route, is very often used to solve the ACVRP problem (Fig. 2).

As a result of combining two or more routes, the total value of the solution is reduced (the authors mention the concept of "saving").

The algorithm envisages the execution of a series of steps in search for an optimal option.

The Clark–Wright algorithm belongs to the approximate iterative methods. Its advantages are its simplicity, reliability and flexibility, which allow for taking into account a number of additional factors, affecting the final solution of the problem. The solution error does not exceed 5-10%.

Disadvantages of this algorithm are: - its efficiency decreases as the end of the calculations is approached; - it takes a lot of time to find a solution, since all the options need to be calculated, stored and sorted out.

In the works of T.J. Gaskell [13], P. Yellow [14], H. Paessens [15] the Clark-Wright algorithm is modified and the above-mentioned flaws eliminated.

One of the classical heuristic transport routing problems (VPR – vehicle performance rating) is the Sweep algorithm [6], [10].

This algorithm is applied in case of polar coordinates and the depot is considered to be the origin of the coordinate system. The depot is then connected to a randomly chosen point, called the "starting point". All the other points are connected to the depot and then
represented by the angles formed by the segments, connecting them to the depot and the segment, connecting the depot to the starting point. The route begins at the starting point, and the points with increasing angles are then included, taking into consideration the given constraints. When a point cannot be included in the route as it would violate a certain constraint, that point becomes the starting point of a new route, and so on. The process is complete when all points are included in the routes (Fig. 3).

In the event that a large number of nodes needs to be served, the Sweep algorithm is used within the “clustered routing” approach. In this case, clockwise, the ratio between the cumulative demand and the vehicle load capacity (including all other constraints) should be checked. The point, which cannot be included due to violation of the vehicle capacity or other limitations, becomes the first point in another cluster. Thus, the entire region is divided into clusters (zones). In the next step, the VRP is solved for each cluster separately. Clustering ends when all cluster points are defined (Fig. 4).

It is certain that one vehicle can serve all the points within a cluster. The final solution depends on the choice of a starting point. By changing the location of the starting point, it is possible to generate different vehicle routes. The routes with the minimum total length should be chosen for the final solution.

After analyzing the classical and modern methods for solving the ACVRP problem, an analogy was established regarding the problem of determining the sequence of changing the cutting tool and its transition from one machined section to another in a process of finishing complex surfaces on a triaxial CNC milling machine.

The efficiency of the cutting tool route, considering the mutual interests of all participants in the system, is taken as a criterion for optimization of the general strategy for mechanical processing: - minimum number of tools used; - minimum length of idle moves when passing between sequentially machined sections; - minimum length of idle moves after finding the most profitable complete tool route between all machined sections.

Finding a solution to both the problem of searching for the minimum number of idle moves during the tool transition between two sections while considering the geometry of the part, and the problem of optimizing the sequence of performing the tool transitions is possible by means of using the well-known approaches for solving the vehicle routing problems.
III. RESULT AND DISCUSSION

When searching for the most advantageous sequence of machining all sections with complex surfaces by cutters of one size, while taking into account the tool life parameters, an algorithm can be applied to minimize the idle tool moves during the transitions from one machined section to another. Such a problem can be described by a graph $G = (V, E)$ with multiple vertices $V = \{v_0, v_1, \ldots, v_n\}$ and edges $E\{e\}$ (Fig.6).

The following associations can be made: $v_0$ - position for changing the cutter; $v_1, \ldots, v_n$ - machined sections; $e \in E$ - length of the edges connecting two of the vertices of the graph with a length, equal to the idle-running displacement $C_e$ when changing the machined section. To build the model it is assumed that the time for the cutter to complete the machined section is $T_i$ and the guaranteed tool life is $Q$.

The $m$ routes of the milling cutter at minimum total length of the idle moves are to be found. These routes should start and end at the position of changing the milling cutter $v_0$ and the tool should pass through each section $v_k$ only once. Besides, each section should be completely machined by means of one milling cutter. In addition, the working time of each cutter should not exceed the set tool life period $Q$.

This type of a problem can be solved in two ways: exact and approximate.

The exact way of solving it is applicable and gives an optimal variant in the event that the machined sections are not a big number.

The approximate method of finding a solution is applied in the case of a large number of machined sections. When applying it, the obtained solution approaches the optimal one.

To solve a problem, in which the number of machined sections is below ten, the so-called "Greedy algorithm" can be applied, which belongs to the group of the heuristic algorithms. When the number of machined sections is bigger than ten, the "Adaptive Large Neighborhood Search" (ALNS) can be applied to find the solution to the problem.

A. Greedy algorithm for optimizing the sequence of machining the sections

Greedy algorithm is considered in detail by a number of authors and finds application in various areas of life [16], [17], [18], [19], [20].

This algorithm envisages the execution of the following actions:

1. Description of all possible sequence options for machining the individual sections;
2. Exclusion of the inadmissible variants;
3. Specifying multiple options for a cutter change sequence for each admissible variant;
4. Exclusion of those sequences of cutter change, in which the cutting time exceeds the tool life period;
5. Of the remaining options of cutter change sequences, choosing the ones, in which the number of cutter changes is the minimum;
6. Determining the auxiliary time, spent on idle tool moves for each variant, at a minimum number of cutter changes;
7. Choosing an optimal variant of a cutter change sequence with minimum auxiliary time, spent on idle moves of the milling cutter.

B. ALNS (adaptive large neighbourhood search) for searching a larger area in the process of optimizing the sequence of machining the sections

ALNS is considered in detail by a number of authors and finds application in various areas of life [21], [22], [23], [24].

ALNS represents an approach for synthesizing different methods of dismembering and joining points. The essence of the ALNS consists in "destruction" of the initially executable version of the sequence of processing individual sections and in "restoration" of the destroyed version. As a result, a new variant of the sequence for processing the sections is obtained. The advantage of the ALNS algorithm is the possibility to evaluate the degree of correspondence (relevance) of each of the methods at each stage of the problem-solving process. Each destruction/restoration method is assigned a "weight" which helps to control the frequency with which the method is used in the search process, and the "weight" of a method can be updated stage by stage [8].

To search for an optimal variant, using the ALNS algorithm, the following steps must be performed:

1. The initial variant for the sequence of machining the sections $x$ is built;
2. Assumption is made that the constructed initial version \( x \) is the best, i.e., \( x_0 = x \), and that the weights of the destruction/restoration methods are the same \( \rho^- = (1, \ldots, 1) \), \( \rho^+ = (1, \ldots, 1) \); 
3. The action is repeated until the stop condition is met. For the purpose:

1. Destruction \( d \in \Omega^- \) and restoration \( d \in \Omega^+ \) methods are chosen, using their weights \( \rho^- \) and \( \rho^+ \); 
2. A process of destruction and reconstruction takes place, resulting in a new variant \( x_t = r(d(x)) \); 
3. Evaluation is made to decide if the variant \( x_t \) is acceptable. If this is the case, then \( x = x_t \). The variant \( x_t \) is not accepted if this option has already been realized.

4. The auxiliary times for each variant \( x_t \) and \( x_b \), are compared. If the variant \( x_t \) is better than the available variant \( x_b \) \( t(x_b) > t(x_t) \), then it is assumed that \( x_b = x_t \). The weights of the destruction/restoration methods \( \rho^- \) and \( \rho^+ \) are updated after that.

4. Going back to \( x_b \).

The ALNS algorithm allows to determine the initial executable variant for a sequence of processing individual sections and makes it possible to choose an appropriate method of destruction of this initial variant. After finishing the destruction, the algorithm chooses a method to restore the destroyed version with the aim of obtaining a better one. This process is repeated until the last feasible option gets close to the optimal one (the iteration is terminated when the stop condition is met). As a result, an option for the sequence of processing individual sections is obtained, in which the additional time, spent on idle tool moves, is minimum.

This paper presents a fragment of a research, aimed at increasing the efficiency of triaxial CNC milling machines when milling parts with complex surfaces by reducing the auxiliary time for performing the individual operations, and more precisely by reducing the time for performing idle tool moves. The latter can be achieved, on the one hand, by minimizing the length of the idle tool moves when changing the machined section, and, on the other hand, by finding the most advantageous sequence for machining the sections.

Based on a conducted literature review [25], [26], [27], [28], [29], [30] a method is proposed for determining the optimal route of cutters of one size when moving among one group of sections subject to machining. The method is implemented in two stages:

- at the first stage, an option for minimizing the idle moves of the tool when changing the machined section is sought. The length of each pair of sections, included in a group, is checked. In doing so, it is necessary to determine the position of the local safety zone for every two consecutively processed sections;
- at the second stage, the most advantageous sequence for machining all the sections in a group is determined, taking into account both the tool life period and the minimum-length idle moves of the cutter, determined during the first stage.

To implement the proposed methodology for minimizing the idle moves when milling details with complex surfaces, a programming module (PM) has been developed. The PM is a synthesis of two modules. The first one, PM1, implements the method of minimizing the idle moves when the tool passes from one machined section to another, while the second one, PM2, is based both on the program module PM1 and on the method of searching for the most advantageous milling sequence for all sections, belonging to the group under processing. PM2 implements the mathematical model considered in the present work. It is based on Microsoft Excel and the VPR Spreadsheet Solver application. The working algorithm of the PM2 module is the following:

Step 1: Entering the input data: data, obtained after the implementation of the PM1 module; cutting time when machining each of the sections; idle moves speed; tool life period; time, spent on changing the cutter.

Step 2: Based on the input data and using the means of the PM2 module, the optimal sequence of milling the sections, machined with one size milling cutters at a minimum length of the idle moves is determined.

The confirmation of the effectiveness of the proposed method for automatic determining of the sequence of execution of the technological transition during pure/finishing milling of parts with a complex shape on a triaxial CNC milling machine should be done by comparing results, obtained for the same details after applying a program module CAM of the Unigraphics system and the developed program module PM.

Test details need to be produced, following a compiled control program.

The results of both the virtual modeling and the physical realization of the machining process, which allow to determine the values of the machine time and the auxiliary time, including the time for idle moves and the time for tool change, should be presented in a tabular format. On the basis of the obtained data, an analysis should be carried out and corresponding conclusions drawn up.

The experimental studies were carried out with a test detail, modeled by using NX 11 (Fig.7).

![Fig.7. 3D model of the test piece.](image)
The finish machining of the test details, which is the purpose of the study, was carried out in two ways. In the first case, the sequence of finish milling of the sections was determined by means of the NX 11 system (option 1). In the second one (option 2), the sequence of finish milling of the sections was determined by using the PM optimization program module.

The parameters of the finish milling mode for both variants remained constant: Spindle rotation frequency \( n = 9000 \text{ min}^{-1} \); Tool feed rate \( V_t = 3000 \text{ mm/min} \); Cutting feed \( S_m = 2500, \text{ mm/min} \) – for flat face cylindrical; \( S_m = 1800, \text{ mm/min} \) – for spherical face cylindrical. A 10 mm diameter four-tooth flat face cylindrical cutter, 10 mm in diameter, and a spherical face cylindrical cutter, 10 mm in diameter, were used for the experiments.

The number of individual sections machined by means of a flat face cylindrical cutter with 10 mm diameter and a spherical face cylindrical cutter with 10 mm diameter, is presented in Table 2.

As a result of the finish machining of the surfaces by means of the specified cutters, the details presented in Fig. 8, were obtained.

### Table 2

<table>
<thead>
<tr>
<th>Cutter</th>
<th>Number of sections, machined with cutters of one size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flat face cylindrical, ( D = 10 \text{ mm} )</td>
<td>17</td>
</tr>
<tr>
<td>Spherical face cylindrical, ( D = 10 \text{ mm} )</td>
<td>11</td>
</tr>
</tbody>
</table>

![Fig.8. Test details after finish milling.](image)

![a) detail 1 after finish milling by the variant without optimization](image) ![b) detail 2 after finish milling by the variant with optimization](image)

Tables 3 present the results of the calculations obtained when preparing the control program for processing the test details based on the NX 11 system without using the optimization program module PM (variant 1), as well as the computational data, obtained by means of the algorithm, which allows to optimize the auxiliary time of idle tool moves when milling complex surfaces on triaxial CNC milling machines (variant 2).

### IV. CONCLUSIONS

The use of an adapted algorithm for solving vehicle routing problems to optimize routes by searching for the most profitable variant of a sequence of idle moves for a triaxial CNC milling machine allows to optimize the task in terms of forming a strategy for processing complex surfaces of parts by adherence to the criterion for minimizing the auxiliary time for moving the tool during milling.

The developed algorithm allows to solve the problem of optimizing the idle tool moves for the case of a complex surface, composed of up to 1000 sections. The result, obtained when applying this algorithm, is approximately accurate and with little time, spent on solving the problem.

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